Time Delay Interferometry and Time Scales in the LISA mission

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Abstract— **The three LISA spacecraft constitute an interferometer that aims at the detection of gravitational waves. They are to be launched in 2015, 5 million kilometers apart, in a triangular configuration, inter-connected by two laser links.
In order to reach the gravitational wave detection level, a Time Delay Interferometry (TDI) data pre-processing method has been developed for LISA. It allows to get rid of (most of) the laser frequency and optical bench noises, which are several orders of magnitude larger than the gravitational wave signal. This TDI analysis is carried out in terms of the coordinate time corresponding to the Barycentric Coordinate Reference System (BCRS), TCB, whereas the data at each of the three LISA spacecraft is recorded in terms of each spacecraft proper time. Hence, one must be cautious using the appropriate relativistic time scales transformations. We address the latter problem in the present paper. We also summarize the TDI data pre-processing and TDI ranging which are novel metrology techniques that might be relevant for further satellite constellation space missions.**

I. INTRODUCTION: LISA, A CHALLENGE

Gravitational wave (GW) observation is to be the next window open on the universe, beside optical, infrared, gamma or X-ray observations. It will lead to new insights in the fields of fundamental physics and astrophysics. Indeed, addressing galactic binaries, compact objects orbiting black-holes, black-hole formation, cosmic background fluctuations… GW detectors are to be the next-generation of telescopes.

GWs were predicted by Albert Einstein, but have not been observed directly yet. Binary pulsars provide strong indirect evidence of their existence, though. In the last years, an international network of terrestrial GW detectors of kilometer size were build and started operations in Japan (TAMA 300), in the United States (LIGO) and in Europe (VIRGO, GEO 600). Those search in the frequency range from 10 to 10^4 Hz. In the meanwhile, next-generation interferometers are being developed.

LISA, the Laser Interferometer Space Antenna [8] is a space detector that aims at the detection of GWs in the [10^{-4}, 10^{-1}] Hz frequency band, complementary to terrestrial detectors. The three satellites that constitute LISA will fly in a quasi equilateral triangle, at an inter-distance of about 5 million kilometers. The triangle will rotate about its center of mass and the later will be on an Earth-like orbit, 20 degrees behind Earth, in the gravitational field of the Sun and planets, as illustrated by Figure 1. GWs crossing the LISA constellation are detected trough the induced change in station inter-distance, measured with double laser links (Figure 2).

The joint ESA-NASA mission LISA is innovative. Indeed, the LISA detector can not be pointed towards individual GW sources. Moreover, most of the signals that LISA will study will be present as a continuous spectrum. And the data will accumulate over years. Another challenge to face: laser frequency and optical bench noises are well above (orders of magnitude) the GW detection threshold. To solve this problem, a new metrology technique, the so-called Time Delay Interferometry (TDI) was developed, based on the precise knowledge of the photon time transfer between LISA stations.
LISA has now entered its formulation phase. Owing to the very specific nature of LISA data and of this new TDI data-pre-processing technique, ESA and NASA recommended that the data analysis scheme be tested: the LISA MOCK DATA CHALLENGE [10] brings together the European Data Analysis Science Team –DAST- and the American AMIGO team. Different LISA simulators [4, 9, 18] and teams are thus put to the test. Simulators must take into account all the relevant aspects (orbitography, photon time transfer, time scales,

phasemeter characteristics, different noises, GW source models…) of the LISA mission and model the TDI technique.

When studying what might affect the efficiency of the TDI technique, in paper [2], we investigated *relativistic* modeling of photon time transfer in LISA, using a priori classical ephemeris for LISA spacecraft. In paper [16], we concentrated on gravitational *relativistic* ephemerides of the LISA spacecraft, and how they might affect estimates of photon time transfer. In the present work and in [15], we consider the problem of converting between the proper time scale of each LISA spacecraft, on which LISA data are archived, and the barycentric coordinate time used by the TDI technique to combine data fluxes.

In the following, we first provide a short description of TDI (data pre-processing and ranging), these new metrology techniques, which might be applied to other space missions. We then provide the required proper time versus BCRS time transformation for LISA.

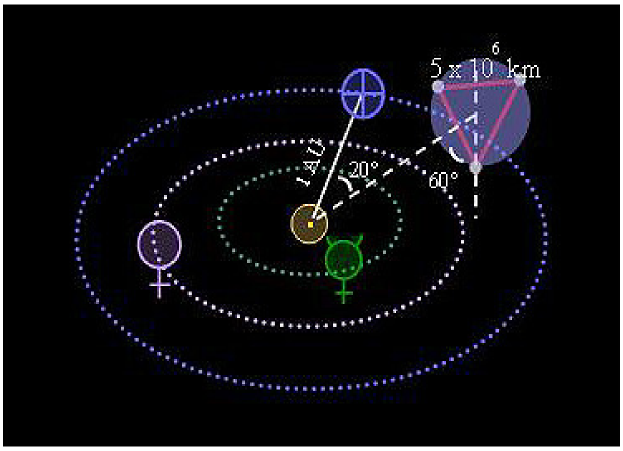

Figure 1. The LISA mission orbit.

II. TDI: NEW METROLOGY METHODS

II.A. TDI data pre-processing

Each of the three satellites of the LISA detector is centered on a free-falling test-mass, the motion of which it follows. GWs are detected through the frequency shift corresponding to the variation ΔL_{ij} they induce in the interferometric inter-distance between the test-mass aboard a satellite *i* and that aboard a satellite *j* of the constellation. To detect such tiny phenomena that are GWs, LISA's arm-length variations must be monitored with high precision: $\Delta L_{ij}/L_{ij} \sim 10^{-23}$ where L_{ij} is the distance between satellites *i* and *j*.

TDI observables are symmetric and delayed combinations of the LISA data fluxes that have the property to cancel the laser frequency (LF) noise and optical bench (OB) noise under certain assumptions. The TDI combinations are functions of the photon time transfer t_{ij} (the delay taken by light to travel the distance L_{ij}). As we shall see, the number of TDI observables available and their efficiency to cancel LF and OB noises depend on the level of modeling of the LISA geometry (i.e. LISA orbitography and time transfer models).

Mathematically speaking, TDI observables can be written as polynomials of time delay operators $E_{ij}(t_{ij})$ applied to data fluxes. The time delay operators delaying any function of time $f(t)$ (a data flux) accordingly: $E_{ij}\,[f(t)]= f(t-t_{ij})$. Under certain assumptions, there is a minimal set of TDI observables called TDI generators which allows writing any TDI observable as a linear combination of those. Let us visualize the different possible data fluxes in LISA as red arrows (plain or dashed) in Figure 2. We can summarize the algebra of TDI operators by distinguishing between 1st, 1.5th and 2nd generation TDI assumptions (See reference [15] and other references therein for a detailed mathematical formulation).

The First Generation TDI assumes constant (in time) and symmetric time delays ($t_{ij}=t_{ji}$); that is, the time for photons to travel from satellite *i* to *j* is the same as to travel from *j* to *i* and it does not change during the whole mission. This leads to p=3 time-delay operators, a minimum number of TDI generators n=4 and a number of different data-flow variables m_{max}=6 (or 9 if the motion of LISA internal parts –such as optical bench motion-, i.e. satellite internal data fluxes are considered). The set of 1st-generation-TDI observables forms the first module of syzygies over a ring of p=3 variables [5].

The 1.5th Generation TDI still assumes constant time delays, but they are not reciprocal anymore ($t_{ij} \neq t_{ji}$). Hence, the number of time-delay operators is doubled (p=6), while n=6 and m_{max}=9. The 1.5th-generation-TDI observables still form a module over a ring of p=6 variables [11].

The 2nd Generation TDI relaxes the constant time-delay assumption ($t_{ij} \equiv t_{ij}(t)$) with respect to the 1.5th Generation TDI. Consequently, the time-delay operators (p=6) do not commute anymore. A new module algebra still must be found. In the meanwhile, in reference [17], the authors propose 6 new, more complex, generalized Michelson- and Sagnac-type 2nd-generation-TDI observables. For those, the order in which the delays ($t_{12}, t_{23}, t_{31}, t_{32}, t_{21}, t_{13}$ if satellites are named 1, 2 and 3) are applied matters.

Let us review the efficiency of TDI observables in removing LF and OB noises, *for ideal, perfect, identical (stable, accurate, without shift nor drift) clocks beating the time t aboard the three LISA satellites and if the time delays* t_{ij} *are known exactly*. Within the 1st-generation-TDI assumptions, the LF (and OB) noises are exactly cancelled in 1st-generation-TDI observables formed from the data-flow variables recorded at each spacecraft. Within the 1.5th-Generation-TDI assumptions, the LF and OB noises are still exactly cancelled by 1.5th-generation-TDI observables. However, when non constant time delays are allowed, even the new 2nd-generation-TDI observables proposed do not remove *exactly* the LF and OB noises. Nevertheless, they bring them down to an acceptable level for LISA. Some further theoretical developments are needed to find better combinations for the 2nd-generation-TDI.

As we shall see in the next paragraph, in the reality, LISA's real geometry is complex and does not fulfill the assumptions of the 1st and 1.5th generations TDI. Consequently, LF and OB noises are not perfectly removed even in the latest, most efficient, TDI generation. The allowed error budget for residual, i.e. post-TDI, LF and OB noises, given by the LISA pre-Phase A Report, is $5 \cdot 10^{-12}$ m/Hz$^{1/2}$. Furthermore, within a same TDI generation, the different TDI observables are not equivalently efficient (see Figure 9 of reference [9] for an illustration). They constitute alternatives in different circumstances; for example, some of them combine all three interferometric arms while some use only two, the latter combinations can still be used if one arm is defective.

In addition to LISA residual LF and OB noises, there are additional inertial test-mass, scattering light effect, beam-pointing, phase measurement and offset lock or detector shot noises… which limit LISA sensitivity curve (see the third column in Table I of reference [9] for the error budget in LISA for these noises, according to LISA pre-Phase A Report, and Figure 6 for an illustration of LISA sensitivity curve, using a TDI-1.5-generation observable, called second generation in that reference).

Now, let us analyze the underlying assumptions of the different TDI generations in the light of LISA geometry (orbitography and laser links) modeling.

The first-generation-TDI assumptions are only met by a rigid, motion-less (this implies that no gravitational bodies are around to cause any motion) LISA constellation model. At this level of modeling, each time delay is given by the corresponding constant interferometric arm-length, slightly different from the generic arm-length $L=5 \cdot 10^9$ m: $t_{ij}=L_{ij}/c$ with $L_{ij} = \|\vec{x}_j - \vec{x}_i\|$, $\vec{x}_i \equiv (x_i, y_i, z_i)$ barycentric coordinates of satellite i and c, the speed of light in vacuum.

The 1.5-generation-TDI assumptions allow for a rotating (around its center of mass and around the barycenter) rigid LISA. At that stage, the Sagnac [12] and aberration effects cause the non reciprocity of time-delays [3]. Indeed, for a same path-length, light rays travelling clockwise and counterclockwise do not take the same time (Sagnac effect). Additionally, there is a motion of the arm-length ij with respect to the barycenter causing an aberration effect. The model for LISA corresponding to 1.5-generation-TDI assumptions consists in *classical* motion (Keplerian) of the three stations in the gravitational field of the Sun up to first order in eccentricity ($e_{LISA} \cong 0.0096$).

The true LISA requires 2nd Generation TDI. Indeed, even a *classical* Keplerian orbital motion around the Sun, if higher orders in eccentricity are considered, causes a flexing of the constellation. $L_{ij}(t)$ is a simple periodic function of t if only the Sun is considered, the so-called breathing of the triangle. If the Newtonian perturbations of planets are considered, the flexing becomes more complex. Moreover, if a gravitational *relativistic* description of photon time transfer is adopted, t_{ij} has additional time-dependant contributions due to gravitational relativistic effects. Indeed, in reference [2, Section III], a native, coherent, gravitational relativistic description of the laser link provides the photon time transfer , as a function of the positions and velocities of the emitting and receiving satellite at emission time: $t_{ij} = t^{(0)}_{ij} + t^{(1/2)}_{ij} + t^{(1)}_{ij}$,
where (m) is the m-th order in $GM/c^2 \sim v^2/c^2$ with G, Newton's constant and M, the solar mass. The 0th order, is the classical time taken by light to travel $L_{ij}(t)$ at velocity c. The 1/2th order contains the Sagnac and aberration effects; and the 1st order, relativistic light deflection or the so-called Shapiro delay. These contributions to the photon flight time were evaluated numerically to $5 \cdot 10^9$ m/c ~ 16.7 s plus a flexing of amplitude of 48000 km/c ~ 0.17 s, ~ $3 \cdot 10^{-3}$ s and < 10^{-7} s respectively, assuming *classical* ephemerides of the three LISA stations.

In reference [16], *relativistic* ephemerides of the three LISA satellites are provided. The authors compute numerically $L_{ij\ relativistic}(t)-L_{ij\ classical}(t)$ and show that it reaches up to ~ 3 km over a year, corresponding to an extra ~10^{-5} s in the 0th order of the photon time transfer.

A realistic model of LISA's orbital motion also leads to geometric (i.e. due to LISA's orbitography and to a relativistic description of photon flight time, not to the presence of GWs) frequency shifts. However, those were shown to be irrelevant to LISA, owing to the mission's detection frequency bandwidth [2, Section V].

With the above discussion in mind, in the context of TDI pre-processing of LISA data, we understand the relevance of realistic models for the orbitography and laser links in LISA.

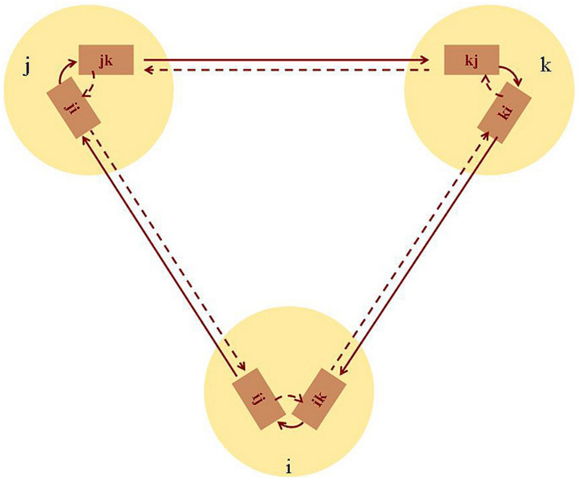

Figure 2. LISA data fluxes model: double laser links between spacecraft *i, j, k*; and double fibre links between the two lasers aboard each spacecraft.

II.B. TDI ranging

Looking at the TDI algebra from another point of view provides an additional interesting metrology tool. The so-called TDI ranging allows us to measure the precise value of the photon time transfer (light-time travelling distance) t_{ij} between two satellites i and j of a constellation. TDI ranging is a minimization process of the values of TDI observables when no gravitational wave is present. Indeed, ideally, if only LF and OB noises are present, all TDI observables corresponding to a module, with the appropriate orbitography model, perfectly cancel for the exact t_{ij} value. Now, in the reality, owing to the true orbitography of LISA and to other noises (see Table I of [9]), the TDI observables are not exactly zero but reach their minimum for the true instantaneous t_{ij} value.

Photon time transfers measured through TDI ranging, $t_{ij\ meas.}$, might then be used as input for the TDI pre-processing of interferometric data. Moreover, with a relativistic model of photon time transfer, from those $t_{ij\ meas.}$, one can access the geometric distance between stations i and j, $L_{ij\ computed}$.

We also note that when TDI-ranging measures residuals with respect to a $t_{ij\ model}$, we have an access to un-modeled noises, to LISA defects, or to fine effects in the gravitational field (for example of the central gravitational body around which the satellite constellation orbits)…

From the above arguments, we understand the need for a relativistic model of laser links (photon time transfer $t_{ij\ model}$), as in reference [2], in the context of TDI ranging, since it provides initial values for a faster optimization process in TDI ranging. Moreover since the laser link model uses a satellite ephemeris for the position and velocity of satellite i and j at photon emission time, we understand the need to investigate a realistic orbitography model, as the relativistic model in reference [16].

III. TIME SCALES IN LISA

In the previous section, we explained the principles of TDI pre-processing. We understand how inaccurate t_{ij}s impact the efficiency of the TDI-preprocessing method which is the prerequisite to gravitational wave detection. A good illustration of this is Figure 8 of reference [9] which shows the consequences of errors of 0, 0.5 or 1 microseconds in t_{ij}s on LISA sensitivity curve for a TDI variable of the 1.5 generation (called 2nd generation in that reference).

In the previous section, we also recalled the relevance of a realistic model of laser links and of an underlying realistic orbitography model for accurate t_{ij}s.

Now suppose that the values of t_{ij}s are somehow accurately known and let us examine the sentence written in the previous section as a condition for perfect cancellation of LF and OB noises: *"ideal, perfect, identical (stable, accurate, without shift nor drift) clocks beating the time t aboard the three LISA satellites"*. In the reality, the clocks (in fact Ultra Stable Oscillators –USO-) aboard the satellites are not perfect. The allowed error budget for USO noise, given by the LISA pre-Phase A Report, is $5\cdot 10^{-12}$ m/Hz$^{1/2}$. These not-so-perfect clocks are used to time-tag the local data fluxes that will be combined in the TDI variables. Hence, we will not use the truly exactly adequate data flux archive (or interpolated data) when implementing whichever TDI variable in any TDI generation, however how theoretically efficient it might be. Consequently, clock errors have the same impact on TDI data-preprocessing efficiency as inaccurate t_{ij}s. So have inadequate interpolation algorithms between archived data flux values.

Reality further adds another trick to clock errors and bad interpolation: Einstein taught us that no absolute physical time scale t exists. In fact, the clocks aboard each satellite k each beat their own proper time $\overset{k}{\tau}$ (*if* those are *not* constrained/synchronized. Indeed, Equation (7) for stations $k=1..3$ orbiting the Sun is similar to the GPS clock synchronization [1] applied to clocks aboard the GPS constellation orbiting the Earth). Local proper time scales of LISA spacecraft $\overset{k=1,2,3}{\tau}$ are used to archive the local LISA data measured at the local phasemeter. The barycentric (TCB) coordinate time t (which is not physical as it can not be measured, only computed) is the time used for photon time transfer in the TDI technique. We understand, thus, how errors when converting between the different time scales in LISA might impact the feasibility of the LISA mission. In the following, we investigate this point and provide the t-$\overset{k}{\tau}$ time transformation.

LISA time scales, are illustrated in Figure 3. Strictly speaking, $\overset{1}{\tau}$, $\overset{2}{\tau}$, $\overset{3}{\tau}$, are different time scales which are not directly compatible, like apples and oranges. The natural common language between those timescales is a time coordinate t such as the one associated with the Barycentric Coordinate Reference System –BCRS-, called TCB. In the LISA mission, the events, which should thus be recorded in the corresponding proper time scale aboard a given LISA spacecraft k, are the different emission or reception event at that spacecraft. The locally measured frequency shifts (against the local proper frequency) should also be regularly recorded on board according to the local proper time scale.

Consequently, to apply the TDI analysis, we need to use equation (7) to convert from time scale $\overset{k}{\tau}$, in which data fluxes are recorded to time scale t.

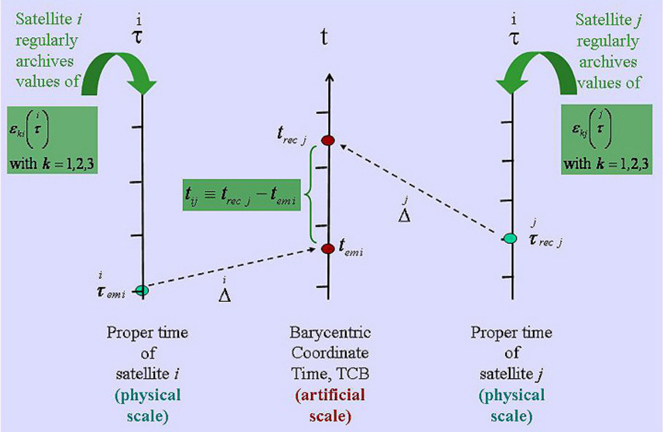

Figure 3. Relativistic time scales in LISA. $\overset{i}{\Delta}$ is the proper time $\overset{i}{\tau}$ of satellite i to BCRS time t transformation; $\bullet_{i\ em}$ and $\bullet_{i\ rec}$ are emission and reception events, respectively, at satellite i; ε_{ki} are data fluxes between satellite k and i that are recorded at i; t_{ij} is the photon flight time from satellite i to satellite j.

III.A. Simple classical orbitography model

To obtain the time transformation between proper time scales of satellites and the TCB time, we model the orbitography of the three stations $(x_k, y_k, z_k)_{k=1,2,3}$ and $(dx_k/dt, dy_k/dt, z_k/dt)_{k=1,2,3}$ with t the BCRS time, *classically*, according to reference [6] in the gravitational field of the sole Sun:

$$\begin{pmatrix} x_k \\ y_k \\ z_k \end{pmatrix} = \Re^{-1} \begin{pmatrix} x_{\mathrm{ell}\,k} \\ y_{\mathrm{ell}\,k} \\ z_{\mathrm{ell}\,k} \end{pmatrix}$$

(1)

with

$$\Re^{-1} \equiv \begin{pmatrix} +\cos\Omega_k \cos\omega - \sin\Omega_k \sin\omega \cos \mathrm{i} & -\cos\Omega_k \sin\omega - \sin\Omega_k \cos\omega \cos\mathrm{i} & +\sin\Omega_k \sin\mathrm{i} \\ +\sin\Omega_k \cos\omega + \cos\Omega_k \sin\omega \cos\mathrm{i} & -\sin\Omega_k \sin\omega + \cos\Omega_k \cos\omega \cos\mathrm{i} & -\cos\Omega_k \sin\mathrm{i} \\ +\sin\omega \sin\mathrm{i} & +\cos\omega \sin\mathrm{i} & +\cos\mathrm{i} \end{pmatrix}$$

and

$$\begin{pmatrix} x_{\mathrm{ell}\,k} \\ y_{\mathrm{ell}\,k} \\ z_{\mathrm{ell}\,k} \end{pmatrix} \equiv \begin{pmatrix} a(\cos\Psi_k - e) \\ a\sqrt{1-e^2}\sin\Psi_k \\ 0 \end{pmatrix}$$

where

$$a = 1\mathrm{AU},$$

$$e = \sqrt{1 + \frac{4}{\sqrt{3}}\frac{L}{2a}\cos\nu + \frac{4}{3}\left(\frac{L}{2a}\right)^2} - 1$$

$$i = \mathrm{arctg}\left(\frac{\frac{L}{2a}\sin\nu}{\sqrt{3/2} + \frac{L}{2a}\cos\nu}\right)$$

and ω are the common semi-major axis, eccentricity, inclination and argument of the periaster of the three spacecraft orbits, respectively. The optimal inclination of the LISA triangle on the ecliptic is $\nu=\pi/3+5/8\cdot L/(2a)$ [13] with $L=5\cdot 10^9$ m, the average interferometric arm-length. The longitude of the ascending node, Ω_k, is particular to a given spacecraft k and is given in terms of that of the first one with a phase shift θ_k:

$$\Omega_k = \Omega_1 - \theta_k$$

with

$$\theta_k \equiv -2(k-1)\frac{\pi}{3}.$$

The time parametrization of the orbits is given by the equation of the eccentric anomaly Ψ_k of each spacecraft,

$$\Psi_k - e\sin\Psi_k = M_k,$$

(2)

with the mean anomaly

$$M_k = \frac{2\pi}{T}(t - t_0) + M_{k0}$$

in terms of the orbital period, T (provided by Kepler's third law, $(2\pi/T)^2 = GM/a^3$, and the mean anomaly of spacecraft k at initial time t_0, that is $M_{k0}=M_k(t=t_0)$. Mean anomalies are related to that of the first spacecraft through the phase shift:

$$M_k = M_1 + \theta_k.$$

The common spacecraft orbit eccentricity being small, the eccentric anomaly equation (2) can be developed at 1st order (with respect to e):

$$\Psi_k \simeq +\frac{2\pi}{T}(t - t_0) + M_{10} + \theta_k + e\sin\left(\frac{2\pi}{T}(t - t_0) + M_{10} + \theta_k\right).$$

(3)

BCRS position and eccentric anomaly equations used in reference [2] correspond to particular initial conditions ($t_0=0$, $\omega=3\pi/2$, $\Omega_1=3\pi/2$, $M_{10}=0$) without any planets (which means that both t_0 and M_{10} are completely arbitrary). When planets are modeled, the LISA guiding center, that is the projection of LISA's center of mass on the ecliptic plane, is supposed to be about 20 degrees behind the Earth at initial time t_0. This means that t_0 and M_{10} are no more arbitrary.

III.B. Corresponding time transformation

We now wish to compute the time transformation between proper time, $\overset{k}{\tau}$, of the clock on board spacecraft k, as a function of coordinate (BCRS) time t, called TCB, since coordinate time is the common "language" between the different spacecrafts (k=1,2,3) and the time used in the TDI method.

If we consider only the gravitational field due to the Sun, the time transformation is given by

$$\mathrm{d}s^2 = c^2\,\mathrm{d}\overset{k}{\tau}{}^2 \simeq \left(1 - 2\frac{w_k}{c^2} - \frac{v_k^2}{c^2}\right)c^2\,\mathrm{d}t^2$$

$$\Rightarrow \mathrm{d}\overset{k}{\tau} \simeq \left(1 - \frac{w_k}{c^2} - \frac{v_k^2}{2c^2}\right)\mathrm{d}t$$

(4)

where $w_k=GM/r_k$ with r_k the radial distance relative to the Sun at time t and v_k the velocity of spacecraft k at time t in the Barycentric Coordinate Reference System.

We can compute the norm of the satellite Keplerian velocity and Keplerian radial distance using equations (1) leading to

$$v_k^2 = \left(\frac{2\pi}{T}\right)^2 a^2 \frac{1 + e\cos\Psi_k}{1 - e\cos\Psi_k}.$$

(5)

$$r_k = a(1 - e\cos\Psi_k).$$

(6)

Substituting those expressions in the time transformation (4), we obtain

$$\overset{k}{\Delta} \equiv \overset{k}{\tau} - t \simeq -\frac{GM}{c^2 a}\int \frac{1}{1 - e\cos\Psi_k}\mathrm{d}t - \frac{a^2}{2c^2}\left(\frac{2\pi}{T}\right)^2 \int \frac{1 + e\cos\Psi_k}{1 - e\cos\Psi_k}\mathrm{d}t$$

$$\simeq -\frac{GM}{c^2 a}\frac{T}{2\pi}\Psi_k - \frac{a^2}{2c^2}\frac{2\pi}{T}\Psi_k - \frac{a^2}{2c^2}\frac{2\pi}{T}e\sin\Psi_k + cst$$

$$\simeq \overset{k}{\tau}_0 - t_0 - \frac{\sqrt{GMa}}{2c^2}\left[3(\Psi_k - \Psi_{k0}) + e(\sin\Psi_k - \sin\Psi_{k0})\right].$$

(7)

The integration constant is given by $\overset{k}{\tau} = \overset{k}{\tau}_0$ and $\Psi_k = \Psi_k(t_0) \equiv \Psi_{k0}$ at initial coordinate time $t=t_0$. The integration was performed through the change in variable $\mathrm{d}t = T\cdot(1 - e\cos\Psi_k)/2\pi\cdot\mathrm{d}\Psi_k$, using the time derivative of the exact expression for the eccentric anomaly (2), and Kepler's third law. From the implicit equation (2)

providing $\Psi_k(t)$, one can compute $\overset{k}{\Delta}$ as a function of time t (Figures 5 and 6). Alternatively, one could use the 1st order expression (3).

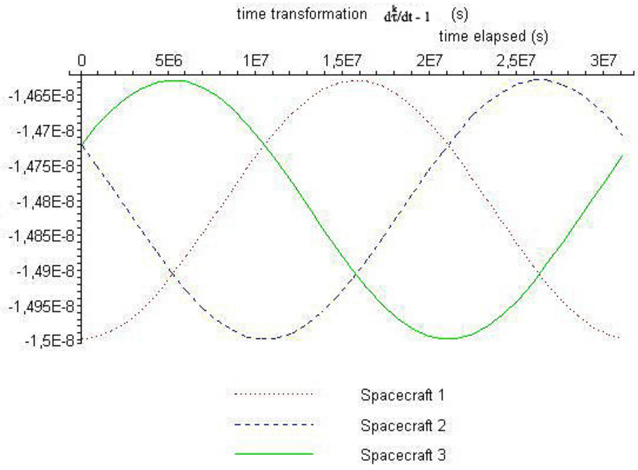

Figure 4. Differential proper versus TCB time transformation for LISA satellite *k*. Formula (4).

To obtain Figures 4, 5 and 6, the initial time was chosen as t_0=0, the initial offsets of the clocks where $\overset{1}{\tau}=0.1$ s, $\overset{2}{\tau}=0.2$ s and $\overset{3}{\tau}=0.3$ s with initial conditions t_0=0, ω=3π/2, Ω_l=3π/2 and M_{10}=0. The cumulative $\overset{k}{\tau}-t$ effect reaches about half a second over one year.

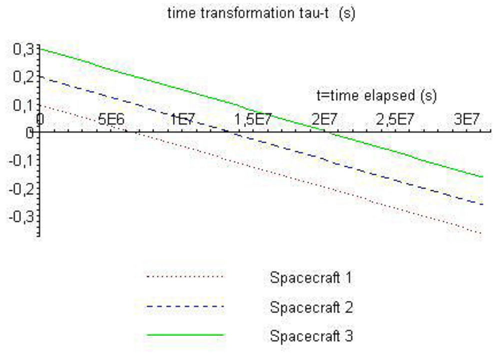

Figure 5. Proper versus TCB time transformation for LISA satellite *k*. Formula (7).

In Figure 6, the linear trend is removed. The amplitude of the $\overset{k}{\Delta}$ oscillatory behavior is ~ 10^{-3} s.

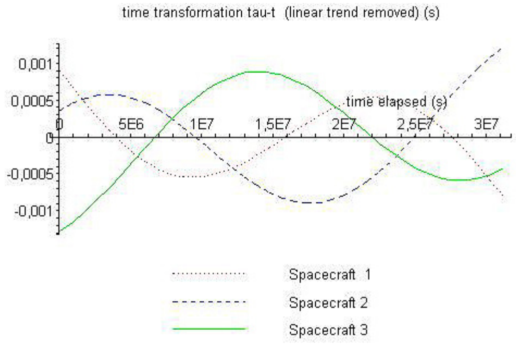

Figure 6. Proper versus TCB time transformation for LISA satellite *k*, linear trend removed in Formula (7).

IV. CONCLUSIONS

To summarize, a coherent realistic (general relativistic) approach of Time Delay Interferometry (TDI) analysis in LISA is necessary. This means general relativistic modeling of laser links such as in reference [2] (that is photon time transfers, since geometric frequency shifts can be omitted owing to LISA frequency bandwidth), LISA station orbitography such as in reference [16] and time scales [15], which is the subject of the present paper.

In LISA, data fluxes at satellites are recorded on the local proper time scale of the constellation satellites, whereas TDI delayed combinations of such data, aiming at reducing laser frequency and orbital bench noises, are function of the Barycentric Coordinate Reference System time, TCB.

We have shown that the difference in rate of spacecraft proper time versus TCB is of the order of $5 \cdot 10^{-8}$. The difference between spacecraft proper times and TCB exhibits an oscillatory trend with a maximum amplitude of about 10^{-3} s. This might be significant in LISA TDI analysis.

To conclude, TDI data pre-processing and TDI ranging are novel metrology techniques that might be relevant for further satellite constellation space missions.

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