Historical measurements of the Sun's diameter variations: some new comments.

Jean-Pierre Rozelot[1], Sophie Pireaux[1], Sandrine Lefebvre[2], and Zahra Fazel[1]

[1] OCA/CERGA
Av. Copernic, 06130 GRASSE, France. email: `rozelot@obs-azur.fr`
[2] Physics and Astronomy Building, UCLA, Los Angeles, CA 90095-1547, USA
email: `lefebvre@astro.ucla.edu`

Abstract. The first pioneer works for measuring the Sun's diameter with an astrometric precision were made around the year 1660 by Gabriel Mouton. Since then, a lot of people (such as Picard, Auzout, La Hire, then Claude, Auwers, Wittman, and many others) have developed more and more sophisticated ground-based techniques. Nowadays dedicated space missions are scheduled for a next future. The question which arises is why we still need to get the Sun's diameter with a very high accuracy. In this paper we will first make an historical review on the question, emphasizing the astrophysical relevant issues. Then we will show how ideas have evolved: today the free surface of the Sun can be defined through a gravitational equipotential that requires a complete knowledge of the Sun's interior. The third part of this paper will be devoted to future programs, such as GAIA, showing how accurate measurements lead to consider, in a relativistic description, the dynamical contribution of solar multipole moments[1].

1 Introduction

Since the highest Antiquity the determination of the value of the solar radius has been a subject widely debated. A number of historical books recount the story and this topic can be considered as ended. By opening a book on Astronomy, such as the *Astrophysical Quantities, 2000*, one may find the value

$$R_\odot = 6.955\,08 \ \pm \ 0.000\,26 \ 10^8 \ \ m$$

which thus appears as a definitive measure of the solar radius because it is widely used today. However, looking carefully at the question, it is not so obvious. First, giving a value of the solar radius requires a definition, as the Sun is not a steel spherical ball. Several expressions can be given. The most commonly accepted is the diameter defined as the distance taken between the two opposite inflection points of the limb intensity profile, at a given wavelength. But other definitions can be used. For instance, an equipotential level of gravity flawlessly defines the limb shape. Secondly, the Sun is a fluid body in rotation. It follows an oblateness

---

[1] Paper presented at the IAGA Assembly, Toulouse (19–29 July 2005), Interdivisional Commission History, Historical Events and People in Aeronomy, Geomagnetism and Solar-terrestrial Physics. To be published by W. Schröder ed., Science edition, Bremen, D.

of the whole figure, and thus the diameter, or more frequently used the semi-diameter (radius), must be identified as equatorial, R_{eq}, or polar, R_{pol}, for which values are (Rozelot and Lefebvre, 2003)

$R_{eq} = 6.95991756\ \ 10^8\ m$ and $R_{pol} = 6.95984386\ \ 10^8\ m$ (uniform rotation),
or
$R_{eq} = 6.95991756\ \ 10^8\ m$ and $R_{pol} = 6.95985961\ \ 10^8\ m$ (non-uniform rotation).

At last, on a pure physical point of view, as the distribution of matter is not uniform inside the Sun, as well as the distribution of the velocity rate, the outer shape shows distortions which are linked to the gravitational moments. Hence, the solar radius, $R(\theta)$, must be a function of the colatitude (θ). As a consequence, all layers that constitute the Sun are not spherical (see Fig. 1). This has been recognized for instance for the tachocline (Charbonneau and Dikpati, 2004).

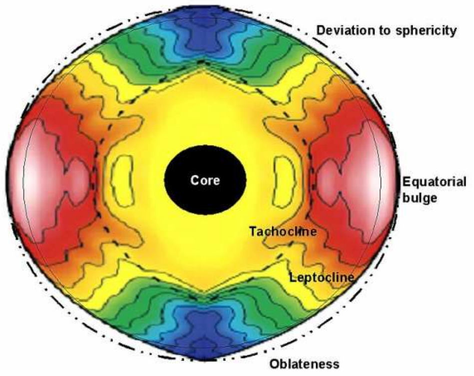


Fig. 1. Due to the non-uniform distribution of mass and velocity rates inside the Sun, the resulting outer shape of the Sun is not spherical and shows deviations to sphericity. However, the global shape remains oblate. The inner dot line (– –) shows the prolate *tachocline* and the thin line (—), the *leptocline*.

The knowledge of the value of the solar radius, once the definition is assumed, is a key parameter not only in stellar physics but also in solar models. On very long-term evolution (several millennia), the solar radius is a function of time. On shorter term (since around 1640 up to now), it has been shown that the solar radius evolves with time too (see Fig. 2 in Pap et al., 2001, or Fig. 3 in Rozelot, 2001, or Fig. 2 and 3 in Rozelot and Lefebvre, 2002, all upgraded from Toulmonde, 1997), likely on a very large periodic modulation, of about 120 years (from one extremum to the other). On shorter periods of time (two or three solar cycles), the temporal variability remained unclear for a long time, but it has been shown recently that it is in antiphase with the solar cycle for layers lying at the very near surface of the Sun, and in phase for layers seated most deeper inside (Lefebvre and Kosovichev, 2005).

The relevance of precise measurements of the Sun's shape can be summarized as followed:

- If R(t) is known to be a function of time over a long period of time ranging over several millennia, and even on undecennal cycles, then luminosity variations can be tackled. However we do not yet know how radius variations on time scales ranging from seconds to hours, if they exist, may play a role in the luminosity variations. Hence, determination, in real time, of the so-called *"asphericity-luminosity parameter"*

$$w = \partial ln(R) \;/\; \partial ln(L)$$

is required. A table summarizing the estimated values of w is given in Fazel et al., 2005a. The knowledge of this parameter is of high importance for the study of the Earth's upper atmosphere.

- If $R(\theta)$ is known, then asphericities coefficients c_n can be deduced, leading in principle to a determination of the solar gravitational moments J_n. The knowledge of these parameters are relevant to celestial mechanics and are required to set up precise ephemeris (due to the relation between J_n and the inclination of the orbits of planets, i.e. spin-orbit couplings) in a General Relativistic description.

- If R(t) and R(θ) are known, then a solar core dynamics can be inferred.

This can be achieved through dedicated space missions, such as GOLF-NG, expected to be launched by 2008-2010 (S. Turck-Chièze et al., 2005).

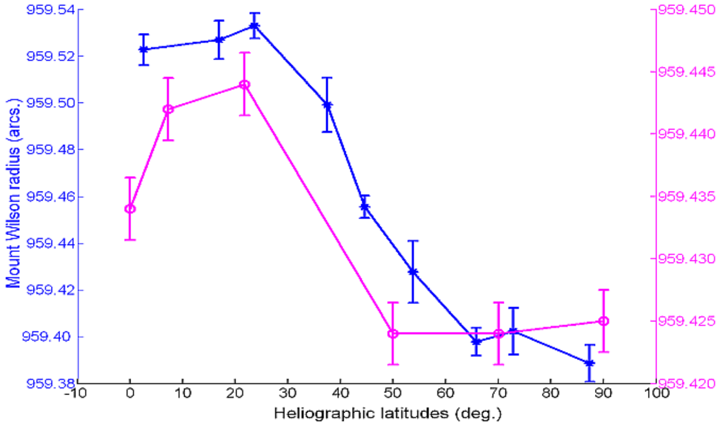


Fig. 2. Comparison between results obtained with the Mount Wilson data, over 30 years of analysis (left scale), and those of the Pic du Midi, obtained on September, 1st, 2001, where exceptional conditions of seeing where encountered (right scale). The solar limb profile does not exactly follow an ellipsoidal shape. (*After Lefebvre et al., 2005).*

2 Observations of the solar shape

The solar shape is very difficult to observe, and hence very difficult to measure with a great accuracy. If Dicke (1970) can be considered as a pioneer in this task, his first attempts at Princeton were not convincing. Several other measurements, made between 1974 and 1994 (see a review in Rösch et al, 1996 or in Pireaux and Rozelot, 2003), lead to more reliable results. Up to that time, only the oblateness was searched for. To summarize, it was shown that, if the Sun were rotating at a uniform velocity rate, the oblateness is[2]

$$\Delta R = (R_{eq} - R_{pol}) = 7\ 370 \quad m \ \text{ or } \quad 10.15 \ \ mas. \tag{1}$$

However, taking the differential rotation into account, the oblateness becomes

$$\Delta R = (R_{eq} - R_{pol}) = 6\ 187 \quad m \ \text{ or } \quad 8.53 \ \ mas \tag{2}$$

It must be noted that the differential rotation decreases the oblateness, in apparent contradiction with the theory of rotating stars. This can be explained by a change in the radial velocity rate near 50° latitude ($d\Omega/dr = 0$ at this latitude, with ($d\Omega/dr > 0$ at higher latitudes and ($d\Omega/dr < 0$ at lower latitudes).

Today, the best results are given through three different techniques yielding satisfactory and coherent results. The first technique, balloon flights and the so-called "SDS" experiment, has been used by S. Sofia. A new description of the experiment and results can be found in Sofia (2005). The second one, still into operation, has been developed at Mount Wilson Observatory. Observations are based on a spectrographic analysis of the neutral iron line Fe I at 525 nm. Measurements have been recently re-analyzed by Lefebvre et al. (2005). The third one is developed at the Pic du Midi Observatory by means of the scanning heliometer, initially conceived by J. Rösch. A full description of the apparatus can be found in Rozelot et al. (1996) and the observational dependence of the solar radius with heliographic latitude is presented in Rozelot et al. (2003). A comparison of the results obtained by these two last techniques is given in Fig. 2 and a full analysis can be found in Lefebvre et al. (2004, 2005). Departures from a pure sphere can be clearly seen: a bulge extends at the equator, up to around 50°, followed by a depression, the polar shape remaining oblate.

Fig. 3 shows two main contributors in the field of the solar shape. As seen before, Dicke has undeniably set the basis of the underlying physics. Even if his papers were often examined critically, they triggered a great amount of ideas which have moved astrophysics forward. Rösch carefully examined the conditions of solar diameter observations, such as blurring effects or displacement of the inflection point toward the inner part of the limb. He defined also the *helioid* as the solar shape, in an analogy with the Earth's *geoid*.

---

[2] "mas" stands for milliarcsecond.
Note that the following value (Eq. 1) is upper-bounded by 10.54 ± 0.25 mas as a maximum and 6.39 ± 1.31 mas as a minimum, according to the value adopted for the uniform velocity rate.

Fig. 3. Left: R. Dicke in its lab, in Princeton. He can be considered as a pioneer for modern measurements of the solar shape (*After Dicke & Goldenberg, Ap. J., 1974*). Right: J. Rösch discussing the electronic devices of the scanning heliometer at the so-called "Coupole Tourelle" of the Pic du Midi Observatory. Above, the book which was edited after a summer school dedicated to his memory (*see ref. Rozelot and Lefebvre, 2003*).

Finally, a recent analysis of the data obtained at the Pic du Midi Observatory shows that the departures of the solar shape from a sphere reach about 20 mas. The oblateness varies slowly in time, in phase with the solar cycle (see figures in Rozelot et al, 2004a or 2004b). A first attempt to understand theoretically solar surfacic distortions was made by Lefebvre and Rozelot (2004) who show that the thermal wind effect is one of the contribution at the solar surface. Note that thermal wind (which is not solar wind) is due to the difference in temperature between the pole and the equator and is the equivalent of the geostrophic effect, well studied by meteorologists.

3 How large are the temporal variations of the solar diameter ?

On physical grounds, temporal variability of the solar diameter cannot exceed 10 *mas* peak to peak in amplitude. Callebaut et al. (2002) were certainly the first to point out that changes in solar gravitational energy, in the upper layers, necessarly involve variations in the size of the envelope. The mechanism is simple. Bearing in mind the definition of the energy $E_g = -\int Gm/r\, dm$ and assuming hydrostatic equilibrium, a thin shell of radius dr (or dm) in equilibrium under the gravitational force and the pressure gradient will expand or contract if any perturbation to these forces occurred. In Fazel et al. (2005b), the authors improved the method and show that any variations of the size of the solar envelope must be within 2.5 *km* of amplitude over a solar cycle, a value in perfect agreement with those deduced from inversion of the f–modes in helioseismology, or from space observations through the MDI data analysis.

Any other larger value is not consistent with astrophysical observations of other solar phenomena. For example, observed temporal irradiance changes, which are observed at a level of ≈ 1 per 1000 over the solar cycle, could be explained by a ≈ 200 mas changes in the solar diameter alone. Such a large value would in turn automatically canceled all other physical explanations and among them, the magnetism of the surface, which is known to explain most (but not all) of the irradiance variation.

As another example, consider for instance the multipolar gravitionnal moments of the Sun. The injection of larger values of $\Delta R(t)$ in models which are tested to other respects, would lead to major impossibilities. Such is the case of the theory of lunar motion for which the inclusion of too large J_n in a spin-orbital motion theory can be accurately confronted to observed lunar physical librations. As these libration are known to a few milliarcseconds of precision, it results that $\Delta R(t)$ is inevitably upper bounded (Rozelot and Bois, 1998).

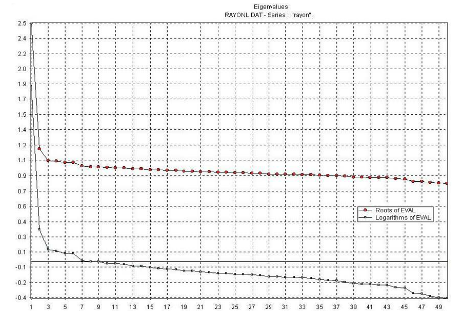
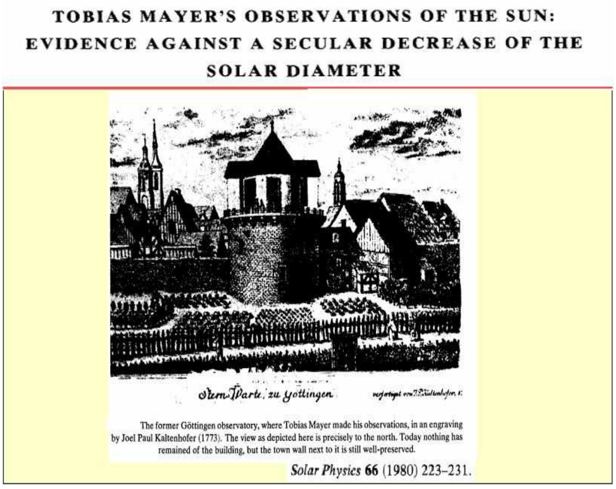


Fig. 4. The Singular Spectrum Analysis applied to the French solar astrolabe data, as an example. It can be clearly seen that only two eigenvalues are detected, the main signal being noise. However, variographic analysis of the same data shows a clear correlation with the stratospheric signal.

The next question the reader may ask is why solar astrolabes, distributed around the Earth (in France, Chile, Brazil and Turkey), are still measuring a diameter variability of about 100 mas, sometimes more. A recent careful analysis, based on a statistical variographic analysis (Badache, 2005) showed that measurements made by astrolabes reflect, without any ambiguity, the fluctuations of the upper Earth atmosphere, i.e. the stratosphere; and *not*, as it is often claimed, the fluctuations of the atmosphere (the turbulence). As the stratosphere is modulated by solar activity (Coughlin, 2005), it results that astrolabes measure a part of the solar signal, but only a small part of it, as the Singular Spectrum Analysis (SSA) shows and as it was suggested earlier (see for instance Pap et al. 2001 and Fig. 4). In fact, one can say that astrolabes are powerful instruments to measure the stratospheric variability and this point is not so trivial.

The last issue is the phase: is the weak solar diameter variability in phase or not with magnetic activity? This question would deserve to be more widely debated, but let us jump to our conclusion, based on three papers: the original paper by Godier and Rozelot (2001), the paper by Fazel et al (2005a) and the paper by Lefebvre and Kosovichev (2005). The first quoted paper describes asphericities in the sub-surfacic layers: one asphericity is located around $0.7R_\odot$ (which is identified as the tachocline), and the another aphericity is located between 0.982 and 0.993 $R_\odot$, with two dips, at 0.986 $R_\odot$ and at 9992 $R_\odot$; this last layer constituting the *leptocline* (see also Fig. 2 in Godier and Rozelot, 2001). The second paper is based on the assumption that the effective temperature of the subsurface is nearly immutable, as suggested from observations made by Livingston at Kitt Peak (2005). It is shown that to model the remaining part of the irradiance variations (not coming from surface magnetism phenomena), there exists a phase-shift in the $[dT, dR]$ plane, with a $dT(dR)$ curve separating solar variations in anti-phase (for temperature values below to 0.08 K), and in phase (for temperature values greater than 0.08 K) with solar irradiance variations. The third one reports changes of the Sun's subsurface stratification inferred from helioseismic inversions. The authors found a variability of the radius in antiphase with solar activity, with the strongest variations just below the surface at about $0.995R_\odot$; the radius of the deeper layers, between $0.975R_\odot$ and $0.99R_\odot$, changing in phase with the 11-year activity cycle. From the above grounds, it can be deduced that the layers located at the sub-surface are the seat of profound physical changes, a fact without any doubt neglected until now.



TOBIAS MAYER'S OBSERVATIONS OF THE SUN:
EVIDENCE AGAINST A SECULAR DECREASE OF THE
SOLAR DIAMETER

The former Göttingen observatory, where Tobias Mayer made his observations, in an engraving by Joel Paul Kaltenhofer (1773). The view as depicted here is precisely to the north. Today nothing has remained of the building, but the town wall next to it is still well-preserved.

Solar Physics **66** (1980) 223–231.

Fig. 5. Measurements of the solar diameter made during the XIX century are discussed by Wittmann in several papers. He showed that the diameter deduced from ground-based observations is $R_\odot = 696\,265 \pm 65\ km$ without any trend.

4 Conclusion

From an historical point of view, the question of whether the diameter of the Sun evolves with time or not is very fertile. On time scales of the order of the millennium, the question of the solar luminosity can be tackled. Models are still needed, but new input will come, paradoxically, from a better knowledge of the temporal evolution of the solar diameter, on smaller time scales. On ranging time going from the medieval era up to now, the debate is not really closed. As Wittman (1980) claims, from Tobias measurements (see Fig. 5), no secular solar diameter decrease can be inferred. We are more in favor of a long-term modulation, the Sun being bigger during periods of lower activity, such as during the Maunder Minimum, and smaller in periods of more intense activity such as presently. On time scales of the order of months, the variability is upper bounded by some 10 *mas*, the Sun being also bigger in periods of strong activity, due to magnetic fluxes passing through the gap between granules without interacting wit them, the photospheric effective temperature playing a key role.

The second issue is the solar changing shape. It has been shown that the outer solar shape significantly differs from a sphere, with a bulge at the equator, and a depressed zone at higher latitudes (the reversal being around 50°, due to the reversal of the radial velocity rate); the whole shape remains oblate at the pole. The Pic du Midi observations show a variability of the whole oblate shape in phase with solar activity (Rozelot and Rösch, 1996, and paper in preparation) that it *is not incompatible* with the above mentioned long-term solar diameter modulation.

Accurate measurements from space observations are needed. They can be achieved by next generation of satellites, such as GOLF-NG (Turck-Chièze et al., 2005), SDO, or even balloon flights (Sofia, 2005). On a longer term, GAIA, which is expected to flight by the end of 2012, will allow to estimate the perihelion precessions of Mercury, Icarus, Talos and Phaeton. In this case, it will be possible to separate the relativistic and the solar contributions in the perihelion advance, so that gravitational moments could be directly determined from dynamics, without the need of a solar model. Note also that presently, dynamical estimates of J_2 are strongly correlated with the estimate of the Post-Newtonian parameter β, which together with other PN parameters, characterizes relativistic theories of gravitation in observational tests. However, future PPN testing space misssions, as well as non dedicated missions like GAIA might help solve the problem.

The problem of determining the temporal diameter evolution of the Sun is still rich and fascinating. We hope to interest a broader community to deeply investigate this field.

Acknowledgments. *This work was partly supported by the French Agency CNRS (UMR 6203). Z. Fazel is partly supported by a grant from the French Ministry of Foreign Affairs and the Research Institute for Astronomy of Maragha in Iran. S. Lefebvre is partly supported by UCLA, the NSF grant ATM-0236682 and the solar physics group at Stanford university. S. Pireaux acknowledges a CNES post-doctoral grant.*

References

1. Allen: 2000. Astrophysical quantities. Fourth edition, Springer ed., p. 340.
2. Badache, C.: 2005, report made at the “Ecole des Mines de Paris”, 71 p. Paper in preparation (A & A).
3. Callebaut, D.K., Makarov, V.I., Tlatov, A.: 2002, ESA SP-477, 209
4. Coughlin, K.: 2005, to be published in “Memorie della Società Astronomica Italiana”, Ermolli, I., Fox, P. and Pap, J. ed.
5. Dicke, R.H. and Goldenberg, H.M.: 1974, Ap. J. S., 241, 27, 131
6. Dicke, R.H. and Goldenberg, H.M.: 1967, Phys. Rev. Letters, 18, 313
7. Dicke, R.H.: 1970, Ap. J., 159, 1
8. Charbonneau, P., Dikpati, M. and Gilman, P. A.: 1999, Ap.J, Issue 1, 523
9. Fazel, Z., Rozelot, J.P., Pireaux, S., Lefebvre, S. and Ajabshirizadeh, A.: 2005a, to be published in “Memorie della Società Astronomica Italiana”, Ermolli, I., Fox, P. and Pap, J. ed.
10. Fazel, Z., Rozelot, J.P., Pireaux, S., Lefebvre, S. and Ajabshirizadeh, A.: 2005b, Sol. Phys., 228, 526
11. Godier, S. and Rozelot, J.P.: 2001, Proceedings of the 11th Cambridge Workshop on Cool Stars, Stellar systems and the Sun, Tenerife, Spain, 4-8/10/1999, CD 649, 6p.
12. Coughlin, K.: 2005, to be published in “Memorie della Societ Astronomica Italiana”, Ermolli, I., Fox, P. and Pap, J. ed.
13. Lefebvre, S. and Kosovichev, A.: 2005, Ap. J. L., submitted.
14. Lefebvre, S. and Rozelot, J.P.: 2004, A & A , 419, p. 1133-1140.
15. Lefebvre, S., Bertello, L., Ulrich, R.K., Boyden, J.E. and Rozelot, J.P. : 2004, “Solar radius measurements at Mount Wilson, SOHO 14/GONG 2004 Workshop, New Haven, ESA SP-559, p. 606-610.
16. Lefebvre, S., Bertello, L., Ulrich, R.K., Boyden, J.E. and Rozelot, J.P. : 2005, Ap. J., submitted.
17. Pap, J., Rozelot, J.P., Godier, S. and Varadi, F.: 2001, A & A, 372, 1005-1018
18. Pireaux, S. and Rozelot, J.P. : 2003, Astrophysics & Space Science, Vol. 284, 1159-1194.
19. Rozelot, J.P. and Rösch, J.: 1996, “Le Soleil change-t-il de forme?”, C.R. Acad. Sci. Paris, 322, 637
20. Rösch, J., Rozelot, J.P., Deslandes, H. and Desnoux, V.: 1996, Solar Physics, 165, 1-11.
21. Rozelot, J.P. and Bois, E.: 1998, “New results concerning the solar oblateness and consequences on the solar interior”, in proceedings of the 18th NSO Workshop, Sacramento Peak, USA, Balasubramaniam ed., 140, 75-82.
22. Rozelot, J.P. and Lefebvre, S.: 2002, in “Solar Variability and Geomagnetism”, Lectures from the IAGA Assembly in Hanoi, 2001, ed. W. Schröder, AKGGKP, Bremen-postdam, pp.88-102
23. Rozelot, J.P. and Lefebvre, S.: 2004. IAU Symposium No.215: Session III: Rotation, solar and stellar physics, Cancun (Mexique, 2002). Edited by A. Maeder and P. Eenens, 2004, p.332-333
24. Rozelot, J.P., Lefebvre, S. and Desnoux, V.: 2003, Sol. Phys., 217, 39-52
25. Rozelot, J.P. and Lefebvre, S.: 2003, in “The Sun’s surface and subsurface”, LNP, 599, Springer ed.
26. Rozelot, J.P., Pireaux, S., Lefebvre, S. and Ajabshirizadeh, A.: 2004, SOHO 14/GONG 2004 Workshop, New Haven, ESA SP-559, p. 532-535

27. Rozelot, J.P., Pireaux, S., Lefebvre, S. and Ajabshirizadeh, A.: 2004, web site adress: astro-ph040382
28. Rozelot, JP: 2001, Jour. Atmosph & Sol. Terres. Phys., 63, 375-386
29. Sofia, S.: 2005, to be published in “Memorie della Società Astronomica Italiana”, Ermolli, I., Fox, P. and Pap, J. ed.
30. Toulmonde, M.: 1997, A & A, 1997, 325, 1174-1178
31. Turck-Chièze, S., Rozelot, J.P. and other co-authors: 2005, “The magnetism of the solar interior, 2005 ESLAB Symposium, “Trends in Space Science and Cosmic Vision 2020, 19-21 April 2005, ESTEC, Noordwijk, The Netherlands. To be published in ESA proceedings.
32. Wittman, A.D.: 1980, Sol. Phys, 66, 223