SF2A 2005
F. Casoli, T. Contini, J.M. Hameury, and L. Pagani (eds)

ON THE KEY ROLE OF A DYNAMICAL ESTIMATE OF THE SOLAR SPIN AND GRAVITATIONAL MULTIPOLE MOMENTS

Pireaux, S.[1] and Rozelot, J-P.[1]

Abstract. We review the question of the solar quadrupole moment (J_2) which is at the crossroad of solar physics, astrometry and celestial mechanics. The order of magnitude of J_2 is known to be 10^{-7}. Precise estimates, however, strongly depend on the method used: stellar equations combined with a differential rotation model, the Theory of Figures of the Sun, or inversion techniques applied to helioseismology. A variability of J_2 with the solar cycle is even considered. Nevertheless, a precise value of J_2 is useful to compute dynamical effects like light deflection in the vicinity of the Sun, or in planetary ephemeris. Conversely, a precise dynamical estimate of J_2 might be crucial to constrain solar density and rotation models. The future space missions GAIA and BepiColombo should lead to a dynamical estimate of J_2, decorrelated from the Post-Newtonian parameter β. A dynamical relativistic Post-Newtonian modeling of planetary motions, including not only the solar quadrupole moment but also its spin, might lead to interesting constraints on those two key solar parameters, via analysis of spin-orbit couplings and of the stability of the Solar System modelled.

1 Definition of gravitational moments and shape asphericities

The outer solar gravitational potential at radial distance r from the solar center (with $r > R_\odot$) is developed on a Legendre polynomial basis (Pireaux & Rozelot 2003). The corresponding coefficients, J_n with n even due to axial symmetry, are the so-called solar gravitational moments.
Shape asphericities, c_n with n even due to axial symmetry, are the coefficients in a development, on a Legendre polynomial basis, of the shape of internal solar layers at radial position r (with $r < R_\odot$) (Rozelot et al. 2004).
Coefficients J_n and c_n are related. Indeed, the solar surface is an equipotential with respect to the total solar potential (i.e. the sum of the gravitational, rotational and magnetic potentials).

[1] Observatoire de la Côte d'Azur, UMR GEMINI, Av. N. Copernic, 06130 Grasse, France

© EDP Sciences 2004

2 Present estimates of J_2

2.1 Different methods

Today, the scientific community agrees on the order of magnitude of the solar quadrupole moment, J_2. However, regarding a precise estimate, different methods yield different values. Let us illustrate this point. Stellar structure equations combined with a differential solar rotation model might provide $J_2 \sim 1.6\,10^{-7}$ (Godier & Rozelot 1999), a value sensitive to the rotation model adopted. The theory of the Solar Figure, an analogy to the geodesical approach for the Earth gravitational field, leads to higher values, $J_2 \sim 6.5\,10^{-7}$ (Rozelot, Lefebvre, 2003). Estimates through helioseismology yield $J_2 \sim 2.2\,10^{-7}$ (Pijpers, 1998). A detailled review of solar quadrupole moment estimates is given in (Pireaux & Rozelot 2003) or (Rozelot et al. 2004).

2.2 Solar cycle dependency

Possible variation of the solar outer shape during the solar cycle was originally proposed by (Dicke et al. 1987). Their controversial 11.14 year-period sinusoidal fit through four data points motivated early solar shape monitorings. The data set has increased drastically in the past years (Badache-Damiani & Rozelot 2005) but we still cannot conclude on the amplitude or on the phase of solar shape variations.

However, the c_2 curve with respect to solar depth (Armstrong & Kuhn 1999; Lefebvre and Kosovichev 2005) reveals two anomalies which identifiy the position of particular solar layers: the leptocline, at the surface, and the tachocline. Observations have shown that the latter is solar cycle dependant.

Furthermore, a recent study on solar irradiance variation with the solar cycle (Fazel et al. 2005) has shown that if one assumes that most ($\sim 95\%$) of the variation can be explained by surface magnetic activity, the rest could be explained by solar shape variations. Nevertheless, the authors could not yet conclude on the phase of such shape variations, due to high sensitivity of the model to small effective solar surface temperature variations.

3 Relevance of J_2 to solar astrophysics and other fields

3.1 Solar astrophysics

From the point of view of solar astrophysics, a number of questions are pending about solar global properties. What is the influence of solar core dynamics on the values of J_n, c_n and global solar spin $\vec{J_\odot}$? What is the influence of solar latitudinal rotation on J_n, c_n and $\vec{J_\odot}$? How to reconcile different estimates of J_n? Do the J_n, c_n and $\vec{J_\odot}$ vary with the solar cycle?...

A precise knowledge of J_n, c_n and $\vec{J_\odot}$ might be crucial to constraint solar models (differential rotation law, density inhomogeneities) or solar evolution.

The solar parameters J_n, c_n and $\vec{J_\odot}$ are not only relevant to solar astrophysics, but their dynamical consequences in relativistic astrometry and celestial mechanics might help set constraints on solar models.

3.2 Relativistic astrometry

Space-time is shaped by the presence of Solar System bodies and the corresponding space-time curvature induced by the Sun leads to light deflection or corresponding time-delays in the propagation of signals. Precise astrometry in the solar neighbourhood will thus require precise knowledge of the solar quadrupole moment and spin. Indeed, in addition to the solar mass monopole contribution (~ 1.75 arcsec at grazing incidence), there is a quadrupolar ($\sim 0.4 - 0.3$ μarcsec at grazing incidence) and differential spinorial ($\sim \pm 0.7$ μarcsec at grazing incidence) solar contribution (Pireaux 2002). Unfortunately, the order of magnitude of the J_2-term drops dramatically as the angle of incidence increases (non-grazing incidence) (Pireaux 2002). At first Post-Newtonian order, the only post-Newtonian parameter present in the light deflection angle expression is γ (with $\gamma \equiv 1$ in General Relativity (GR)). It encodes the amount of curvature of space-time per unit rest-mass.

3.3 Relativistic celestial mechanics

The solar quadrupole moment also plays a role in celestial mechanics. Its direct influence on the relativistic precession of the perihelion of planets is well known (Pireaux & Rozelot 2003), $\sim$ 43 arcsec in the case of Mercury. Together with γ, the post-Newtonian parameter β also contributes to the relativistic precession. The latter parameter encodes the amount of non-linearity in the superposition law of gravitation (with $\beta \equiv 1$ in GR).
A precise knowledge of J_2 might be usefull for precise ephemeris. Indeed, presently, there is a strong correlation between β and J_2 in planetary ephemeris, up to 80% for some data sets (Standish 2004-2005). Hence, one cannot fit simultaneously for those two parameters. Furthermore, the solar quadrupole moment has other direct influences, as on planetary spins and on the ecliptic plane. A better knowledge of J_2 would thus help in long-term Solar System modelization (Laskar 1999; Laskar et al. 2004). Finally, through Solar System spin-orbit couplings, J_2 and $\vec{J_\odot}$ will indirectly influence the orbital parameters of Solar System bodies. For example, the Moon-Earth spin-orbit coupling propagates the influence of the solar quadrupole moment to the Moon. This allowed to set a dynamical upper bound on the solar quadrupole moment, $J_2 \leq 3\,10^{-6}$ (Rozelot & Rosch 1997; Bois & Girard 1999), through observed lunar librations.

3.4 Tests of alternative theories of gravitation

We have seen that presently the β and J_2 parameters are correlated. Using a reasonnable value for J_2 (Pireaux & Rozelot 2003) and present best constraints on post-Newtonian parameters γ and β (Bertotti et al. 2003; Williams et al.

2002, respectively), we showed that GR is still in the battle, but there is room for alternative theories too (Rozelot et al. 2004).

4 Future space missions

Future solar space missions like GOLF-NG (2006 and 2010-2012 for the ground- and space-based prototypes respectively) and SDO (Solar Dynamics Observatory, 2008-2013) should improve our knowldge on solar diameter, rotation and core dynamics. The microarcsec-astrometric mission, GAIA (2012), and Bepi-Colombo (2011-2012), plus dedicated missions should set stronger constraints on post-Newtonian parameters. GAIA should be able to decorrelate β and J_2 thanks to a sampling in the (a, e, i)-planetary parameter space from the precession of minor planets. BepiColombo should measure the precession of the orbital plane of Mercury around the polar axis of the Sun, due to J_2.

5 A "Relativistic Spin-Orbit Coupling and Solar Astrodynamics" project

SONYR (Spin Orbit N-body Relativistic model) is a tool developed by E. Bois and N. Rambaux. This N-body model integrates, relativistically, the motion of Solar System bodies in the Damour-Soffel-Xhu formalism (up to first post-Newtonian order), in the setting of GR. SONYR includes tidal deformations of planets, planetary multipolar figure, the solar spin and quadrupole moment. The idea is to use SONYR to infer dynamical relativistic constraints on J_2 and $\vec{J_\odot}$, using the spin-orbit coupling properties of the Solar System. The stability of the Solar System modelled could be analysed and the modelled planetary orbital parameters could be confronted with observations. Furthermore, a version of SONYR including post-Newtonian parameters might provide a test of alternative theories of gravitation.

References

Armstrong, J. & Kuhn, J. R., 1999, Astrophys. J., 525, 533

Badache-Damiani & Rozelot, 2005, in preparation

Bertotti, B., Iess, L. & Tortora, P., 2003, Nature, 425 , 374-376

Bois, E., & Girard, J-F., Cel. Mech., 1999, 73, 329-338

Dicke, Khun & Libbrecht, 1987, Astrophys. J., 318, 451-458

Fazel, N., Rozelot, J-P., Pireaux, S., Lefebvre, S. & Ajabshirizadeh, A., June 2005, submitted to Solar Phys.

Godier, S. & Rozelot, J-P., 2001, Solar Phys, 199, 227

Laskar, J., 1999, Phil. Trans. of the Royal Society of London A, 357, 1735-1759

Laskar, J., Robutel, P., Joutel, F., Gastineau, M., Correia, A. C. M. & Levrard, B., 2004, A&A, 1335Lask, http://hal.ccsd.cnrs.fr/ccsd-00001603

Lefebvre, S. & Kosovichev, A., Astrophys. J., submitted 2005

Pireaux, S., 2002, Light deflection experiments as a test of relativistic theories of gravitation, PhD. Thesis, LLN, Belgium, 2002.

Pireaux, S. & Rozelot, J-P., 2003, Astrophys. and Space Sc., 284, 1159-1194

Rozelot, J.P. & Rösch, J., 1997, Solar Phys., 172, 11-18

Rozelot, J.P., Pireaux, S. & Lefebvre, S., 2004, astro-ph/04033082

Standish, E. M., 2004, private communication

Williams, J. G., Boggs, D. H., Dickey, J. O. & Folkner, W. M., 2002, Proceedings of the 9th Marcel Grossmann 2000, World Scientific, 1797-1798