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# PROPER TIME VERSUS TCB USED FOR TIME DELAY INTERFEROMETRY IN THE LISA MISSION

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**Abstract.** The three spacecraft $i, j, k = 1, 2, 3$ of the LISA (Laser Interferometer Space Antenna) joint ESA-NASA mission aim at the detection of gravitational waves (GW). They are to be launched in 2015, $L_{ij} \sim 5 \cdot 10^6$ kilometers apart, in a triangular configuration, each following a free-falling test mass. The test masses are inter-connected by double laser links forming an interferometer. Laser frequency (LF) and optical bench (OB) noises are several orders of magnitude larger than GW signals to be detected. Hence, Time Delay Interferometry (TDI) data pre-processing (summarized here) was developed to reach the gravitational wave detection level, allowing to get rid of (most of) LF and OB noises. TDI combination algebra, to be applied on the data, is given in terms of the coordinate time, $t$, corresponding to the Barycentric Coordinate Reference System (BCRS) and called TCB. However, local data at each spacecraft is recorded in terms of spacecraft proper time, $\overset{k}{\tau}$, requiring the use of relativistic time transformations $\overset{k}{\tau} - t$ provided here.

## 1 LISA, a challenge

LISA (LISA, 1998) will detect GW in the $[10^{-4}, 10^{-1}]$ Hz frequency band. A GW is a space-time perturbation. When passing by, it perturbs the free-falling test-mass motion. GWs are detected via the measurement of phase shift due to the interferometric arm-length variations $\Delta L_{ij}$. This requires LISA arm-length to be known with a good precision: $\Delta L_{ij}/L_{ij} \sim 10^{-23}$. In LISA, LF and OB noises are orders of magnitude larger than GW signals ($\sim 10^{-13}$ versus $\sim 10^{-21}$ in fractional frequency units). However, through TDI data pre-processing, LF and OB noises can be reduced by more than 8 orders of magnitude. TDI pre-processing is based on the precise knowledge of photon flight time $t_{ij}$ between two LISA spacecraft, while TDI ranging allows to measure $t_{ij} = L_{ij}/c + ...$ with $c$, the speed of light in vacuum.

## 2 TDI: a new metrology method

LISA has 6 independent laser links, leading to 12 interferometric measurements (inter or intra-satellites). TDI observables are time-delayed (with respect to $t_{ij}$) particular combinations of measurements from different laser links, in close loops, in which OB and LF noises are cancelled. Mathematically speaking, TDI observables are polynomials of delay operators that are applied on measurements. The TDI algebra (Nayak & Vinet 2005, Shaddock *et al* 2003) is based on TDI generators, that is a set of TDI observables allowing to write any other as a linear combination of those; and is characterized by the numbers $p$, $n$, and $m_{max}$, defined in Figure 1 Left. Those numbers are function of the selected set of assumptions on $t_{ij}$, called 1st, 1.5th or 2nd generation (Figure 1 Left). The latter assumptions are verified only by certain orbit models (Figure 1 Middle).

On one side of the problem, the appropriate geometry model for LISA and TDI generation must be selected so that, through *TDI data pre-processing*, LF and OB noises are brought down to LISA specifications. Ideally, when only LF and OB noises are present and when the efficiency assumptions (Figure 1 Right ) are verified, the efficiency of the TDI generations to remove LF and OB noise is optimal. It is however not a 100 % for the 2nd generation TDI, since the algebra for that generation is not exact. Furthermore, for the real LISA mission, there will be other noises than LF and OB, the true orbits will be more complex than the model verifying TDI 1st, 1.5th or 2nd generation assumptions, and the efficiency conditions will not be verified... thus lowering TDI

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efficiency. In particular, the clocks (Ultra Stable Oscillators, USOs) aboard the 3 spacecraft do not beat the coordinate time $t$ used in TDI. This is the toppic of the present paper and it is addressed in Section 3. On the other side, *TDI ranging* measures $t_{ij}$ via a minimization of residual LF/OB noise in TDI observables as, ideally, if only LF and OB noises are present and efficiency conditions are verified, the TDI-observables cancel for the appropriate $t_{ij}$.

TDI metrology requires a realistic geometry, that is laser link $t_{ij}$ model (Chauvineau *et al* 2005), orbital velocity $v_k$ and position $r_k$ models for spacecraft $k$ (Pireaux & Chauvineau 2008); possibly including relativity.

## 3 Relativistic time scales in LISA

The clock abord spacecraft $k$ beats its own proper time $\overset{k}{\tau}$ used to timestamp the data, not $t = TCB$ used in TDI (Pireaux 2007, in particular Figure 1 ). To find the time transformation $\overset{k}{\tau} - t$, we use the relativistic line element for a BCRS metric, leading to

$$ds^2 = c^2 d\overset{k}{\tau}{}^2 \simeq \left(1 - 2\frac{w_k}{c^2} - \frac{v_k^2}{c^2}\right) c^2 dt^2 \;\Rightarrow\; \overset{k}{\tau} - t \simeq \overset{k}{\tau}_0 - t_0 - \frac{\sqrt{GMa}}{2c^2}\left[3\left(\Psi_k - \Psi_{k0}\right) + e\ \left(\sin\Psi_k - \sin\Psi_{k0}\right)\right]$$

where we used a simple classical Keplerian orbit model (Nayak *et al*, 2006) with eccentric anomaly $\psi_k$, semi major axis $a$, Newtonian constant $G$, solar mass $M$ and Newtonian solar potential at spacecraft $k$, $w_k$. This allows to compute numerical estimates for a one year mission as in Figure 2 (Pireaux 2007). We see that the difference in rate of spacecraft proper time versus TCB is of the order of $5 \cdot 10^{-8}$. The difference between spacecraft proper times and TCB exhibits an oscillatory trend with a maximum amplitude of $\sim 10^{-3}$ s.

| Characteristics of the TDI algebra | TDI generations | | |
|---|---|---|---|
| | 1st | 1.5th | 2nd |
| delay assumptions | $t_{ij} = t_{ji}$, $t_{ij} = cst$ | $t_{ij} \neq t_{ji}$, $t_{ij} = constant$ | $t_{ij} \neq t_{ji}$, $t_{ij} \neq constant$ |
| TDI algebra | 1st module sysyguies over ring p=3 | 1st module sysyguies over ring p=6 | ? |
| $m_{max}$ = nbr of different data-flow variables | 6 or 9 | 9 | 9 |
| $p$ = nbr of delay operators | 3 | 6 | 6 non-commutative |
| $n$ = nbr of TDI generators in the minimal set | 4 | 6 | 6 TDI to keep LF/OB within specifications but no algebra |

| Appropriate geometry model to fit delay assumptions | TDI generations | | |
|---|---|---|---|
| | 1st | 1.5th | 2nd |
| delay assumptions | $t_{ij} = t_{ji}$, $t_{ij} = cst$ | $t_{ij} \neq t_{ji}$, $t_{ij} = constant$ | $t_{ij} \neq t_{ji}$, $t_{ij} \neq constant$ |
| orbit model | rigid, motionless LISA | rigid, rotates ↻ CM, 1st order in $e$ Keplerian motion ↻ Sun | breathing of constellation |
| laser link model | Newtonian: $t_{ij}=L_{ij}/c$ | Newtonian + Sagnac +aberration effects | Newtonian + Sagnac +aberration + relativity effects |

| TDI efficiency to remove LF/OB noises | TDI generations | | |
|---|---|---|---|
| | 1st | 1.5th | 2nd |
| delay assumptions | $t_{ij} = t_{ji}$, $t_{ij} = cst$ | $t_{ij} \neq t_{ji}$, $t_{ij} = constant$ | $t_{ij} \neq t_{ji}$, $t_{ij} \neq constant$ |
| efficiency assumptions: - 3 ideal, identical, perfect spacecraft clocks (USO) - 3 clocks beating $t$ - $t_{ij}$ exactly known | 100% | | <100% |
| if efficiency assumptions not verified | <100% | | |

**Fig. 1.** Characteristics of TDI. See text and (Pireaux 2008) for details.

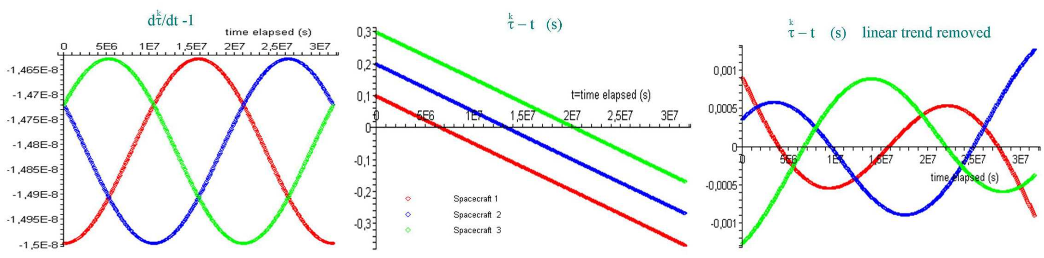


**Fig. 2.** Proper versus coordinate time in LISA, over one year. See text and (Pireaux 2007) for details.

## References


Chauvineau, B., Pireaux, S., Regimbau, T., Vinet, J-Y., 2005, Phys. Rev. D, 72, 122003

LISA Pre-Phase A Report, 2nd Ed., 1998: http://www.srl.caltech.edu/lisa/documents/PrePhaseA.pdf

Nayak, K.R., Vinet, J-Y., 2005, CQG, 22, S437-S443

Nayak, K.R., Koshti, S., Dhurandar, S.V., Vinet, J-Y., 2006, CQG, 23, 1763-1783

Petiteau *et al*, 2008, Phys. Rev. D, 77023002

Pireaux, CQG, 24, 2271-2281, 2007

Pireaux, proceedings of EFTF08, Toulouse, France, 23-25 March 2008

Pireaux, S., & Chauvineau, B., Cel. Mecha., submitted 2008; arXiv:0801.3637

Shaddock, D.A., Tinto, M., Estabrook, F.B., Armstrong, J.W., 2003, Phys. Rev. D, 68, 061303(R)