INTEGRATING THE MOTION OF SATELLITES IN A CONSISTENT RELATIVISTIC FRAMEWORK: THE SCRMI PROTOTYPE SOFTWARE

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ABSTRACT

The "Newton plus relativistic corrections" orbitography software now in wide use faces three major problems. First of all, they ignore that in General Relativity time and space are intimately related, as in the classical approach, time and space are separate entities. Secondly, a (complete) review of all the corrections is needed in case of a change in conventions (metric adopted), or if precision is gained in measurements. Thirdly, corrections can sometimes be counted twice (for example, the reference frequency provided by the GPS satellites is already corrected for the main relativistic effect), if not forgotten. For those reasons, a new native relativistic approach is suggested. In this relativistic approach, the relativistic equations of motion are directly numerically integrated for a chosen metric. Our prototype software, that takes into account non-gravitational forces, is named SCRMI (Semi-Classical Relativistic Motion Integrator).

INTRODUCTION: THE CLASSICAL APPROACH

Today, the motion of spacecrafts is still described according to the classical Newtonian equations plus the so-called "relativistic corrections", computed with the required precision using the Post-(Post-) Newtonian formalism (ref. 1), as described by equation (1).

$$\frac{d^2X^i}{dT^2} = -\frac{\partial W}{\partial X^i} - K^i + \textit{general relativistic corrections.} \quad (1)$$

where T is the coordinate time, W is the gravitational potential (including the central planetary potential model but also the Earth-Tides (solid and liquid) potential and planetary and Sun Newtonian potentials), and K is the non-gravitational perturbation (atmospheric drag, the radiation pressure, etc…). This classical approach, with the increase of tracking precision (Ka-Band Doppler, interplanetary lasers) and clock stabilities (atomic fountains) is reaching its limits in terms of complexity, and can be furthermore error prone.

THE SEMI-CLASSICAL RELATIVISTIC APPROACH: SCRMI

In the more appropriate framework of General Relativity, it is well known that when considering gravitational forces alone, the unperturbed satellite motion follows the geodesics of the local space-time (ref. 2). When non-gravitational forces are present, the relativistic equation of motion generalizes to include non-gravitational forces now encoded in a quadrivector K as

$$\frac{dU^\alpha}{d\tau} = -\Gamma^\alpha_{\beta\gamma} U^\beta U^\gamma + K_\beta \left(G^{\alpha\beta} - \frac{U^\alpha}{c}\frac{U^\beta}{c} \right) \quad (2)$$

$$\text{with } U^\alpha \equiv \frac{dX^\alpha}{d\tau}, \quad U^\alpha U_\alpha = c^2 \quad (3)$$

where $X^{\alpha=0,1,2,3} \equiv (c \cdot T,\ X^i)$ are the space-time coordinates; $\Gamma^\alpha_{\beta\gamma}$ are the Christoffel symbols; $G^{\alpha\beta}$, space-time metric; c is the speed of light; and τ is the proper time. When $K_\beta = 0$, equation (2) reduces to the geodesic equation of the local space-time.

For the appropriate metric at the required order, this equation contains all the gravitational effects at the corresponding order. Non-gravitational forces can be treated as perturbations, in the sense that they do not modify the local structure of space-time (the metric). Moreover, K being small, one can safely replace the metric tensor G by its Minkowskian counterpart in the second term of the right-hand-side of equation (2), hence the terminology “Semi-Classical”.

Equation (2) is at the core of SCRMI (Semi-Classical Relativistic Orbitography Software), and is computed with respect to the Geocentric Coordinate Reference System (GCRS) metric (ref. 3 and ref. 4). It takes into account gravitational multipole moment contributions from the central planetary gravitational potential, perturbations due to solar system bodies, the Schwarzschild, geodesic and Lense-Thirring precessions.

Expression (2) consists of four equations, to be compared with the three equations (1) to be integrated in Newtonian mechanics. However, a first integral exist, i.e. equation (3), since the norm (with respect to the metric G) of the quadri-proper-velocity U is conserved along the trajectory. This point stresses the importance to use a symplectic integrator to naturally preserve this quantity.

Such an integrator can be found in the Runge-Kutta family. If we consider a s-stage Runge-Kutta p-order method of the form

$$y_{n+1} = y_n + h\sum_{i=1}^{s} b_i\, k_i \quad (4)$$

$$with\; k_i = f(y_n + h\sum a_{ij}\, k_i) \quad (5)$$

for the differential equation $\dot{y} = f(y)$, where h is the time step, $y_n = y(t_n)$ and $y_{n+1} = y(t_n + h)$, then the method is symplectic if and only if the Runge-Kutta coefficients b_i and a_{ij} characteristic of the method satisfy (ref. 5)

$$b_i\, a_{ij} + b_j\, a_{ji} = b_i\, b_j \quad for\; all\; i, j = 1,\ldots,s \quad (6)$$

CONCLUSION

We have outlined here a new paradigm for orbitography software that is, in our opinion, the key (or at least a key) for the future. The concepts differ radically from those in use today. To implement these ideas requires rewriting the core parts of existing programs; they cannot be simply “upgraded”. This is obviously the main difficulty to overcome, as well as the resistance people have about what is perceived as exotic physics. Full details about the SCRMI approach can be found in ref. 6, 7, 8 and 9.

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