To the readers…

In order to improve these notes, please report any errors that you may encounter (there might be some!) to

sophie.pireaux@cnes.fr

I shall appreciate your comments…

Centre de formation IGN, Forcalquier, 30/08/04 – 04/09/04

Deuxième école d'été du GRGS:

Géodésie spatiale, Physique de la Mesure et Physique Fondamentale

Sophie Pireaux

Observatoire Midi-Pyrénées,
UMR5562 DTP – GRGS (Groupe de Recherche en Géodésie Spatiale)

Observatoire
Midi-Pyrenees

Conventions:

Usual partial derivative with $\frac{\partial \bullet}{\partial x^{\mu}} = \bullet_{,\mu}$

Covariant derivative with $\bullet_{|\mu}$

Latin indexes for 1,2,3 (spatial coordinates)

Greek indexes for 0,1,2,3 (space-time coordinates)

Einstein's convention for indexes summation:

$$\bullet^{\mu}\,\bullet_{\mu} = \sum_{\mu} \bullet^{\mu}\,\bullet_{\mu}$$

Session I. Space Geodesy and relativity

- Mathematical introduction to relativistic theories
- General relativity and scalar-tensor theories
- Key tools for relativistic gravitational effects
- Introduction to relativistic celestial mechanics
- Geodesy and tests of relativistic theories of gravitation
- Light propagation in curved space-time

A/ Key tools for relativistic gravitational effects

- Parametrized Post-Newtonian (PPN) formalism
- Constancy of the Newtonian gravitational constant (G)
- IAU resolutions on reference systems

B/ Introduction to relativistic celestial mechanics

- Two body system in weak field approximation
- Main relativistic effects in celestial mechanics and tests of relativistic theories of gravitation

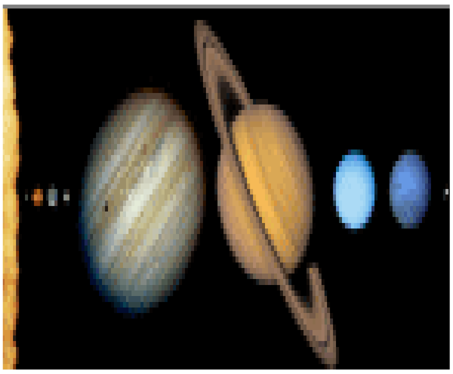
C/ Geodesy and tests of relativistic theories of gravitation

- Geodetic satellite equations of motion
- Main relativistic effects in geodesy
- Satellite tests of relativistic theories of gravitation

B/ Introduction to relativistic celestial mechanics

PPN formalism,
n-body weak field approximation

RELATIVISTIC THEORIES
OF GRAVITATION



B1. Two body system in weak field approximation

- **Weak field *n-body system*, acceleration on body "*a*":**

CANNOT assume a massive self-gravitating body to move along geodesics of the PPN metric because its motion may depend of its internal structure (ex: WEP violation) (but ok for GR).

➡ Use

- Celestial body "*a*" ~ finite self-gravitating clump of matter, assumed to be perfect fluid, with total mass energy defined in local, comoving nearly inertial frame of the body

$$m_a \equiv \int_a \rho^* \left(1 + \frac{1}{2}\bar{v}^2 - \frac{1}{2}\bar{U} + \Pi\right) \cdot d^3x$$

conserved density, kinetic energy, gravitational energy, inertial energy

and center of total mass

$$\vec{x}_a \equiv m_a^{-1} \int_a \rho^* \left(1 + \frac{1}{2}\bar{v}^2 - \frac{1}{2}\bar{U} + \Pi\right) \vec{x} \cdot d^3x$$

- $T^{\mu\nu}{}_{|\nu} = 0$ Conservation of the Energy-Momentum tensor. Describes stress matter and non gravitational fields.

Conservation laws in local Lorentz frame momentarily moving with the matter:

$$\rho^* = \sqrt{-g} \cdot \rho \cdot \frac{dx^0}{d\tau}$$ = conserved density

$\vec{v}$ = baryon velocity

$$\frac{d\rho^*}{dt} + \vec{\nabla}\rho^* \cdot \vec{v} = 0$$

"Eulerian" continuity equation (derived from the conservation of the energy-momentum tensor)

$$\bar{U} \equiv G \int_a \frac{\rho^*(\vec{x}', t)}{|\vec{x}' - \vec{x}|} d^3x$$

Newtonian potential generated by body "a"

➡ Compute $\vec{v}_a \equiv \dfrac{d\vec{x}_a}{dt}$

$\vec{a}_a \equiv \dfrac{d\vec{v}_a}{dt} = \ldots$

and simplify terms by
- substituting Newtonian equations of motion in PPN term
- using continuity equation for ρ^*

➡ Compute $T^{\mu\nu}{}_{|\nu} = 0$ in PPN formalism for a PPN metric to obtain ρ

and reformulate as $\rho^* \dfrac{dv^j}{dt}$ = function of
- ρ^*, p
- PPN potentials
- PPN parameters
- matter velocity
- PPN reference frame velocity
  1/1 mean rest-frame of Universe

➡ Insert $\rho^* \dfrac{dv^j}{dt}$ in $\vec{a}_a$

➡ Assume timescale of structural changes in body "*a*" << timescale of orbit of "*a*"
Average over internal dynamical timescale
- d(internal quantity)/dt ≟ 0
- use Newtonian virial equ. to simplify terms

➡ Results: $\vec a_a = \vec a_a\big|_{self} + \vec a_a\big|_{Newton} + \vec a_a\big|_{n\,body}$ [Will 1993, p149] where $G=c=1$, $\alpha=1$

a) Self-accelerations of the center of mass of body "a", due to its internal structure :

$$a_a^j\big|_{self} = -m_a^{-1}\left[\begin{array}{l} +\frac{1}{2}(\alpha_3+\zeta_1)\,t_a^j + \zeta_1\left(\mathscr{T}_a^j - \frac{3}{2}\mathscr{T}_a^{**j}\right) \\ +\zeta_2\Omega_a^j + \zeta_3\mathscr{E}_a^j + 3\zeta_4\mathscr{P}_a^j \\ -\alpha_3(w+v_a)^k H_a^{kj} \end{array}\right]$$

with $t_a^j, \mathscr{T}_a^j, \mathscr{T}_a^{**j}, \Omega_a^j, \mathscr{E}_a^j, \mathscr{P}_a^j, H_a^{kj}$ vector/tensor integrals over $\rho^*,\ p\ ,\ \vec v\ ,\ \Pi$ in "a" [Will 1993, table 6.2]

in **semi-conservative theories**, $\alpha_3, \zeta_1, \zeta_2, \zeta_3, \zeta_4 = 0 \Rightarrow a_a^j\big|_{self} = 0$

for a **spherically** symmetric body "a" ,
or binary system with (nearly) **circular** orbit, $t_a = \mathscr{T}_a^j = \mathscr{T}_a^{**j} = \Omega_a^j = \mathscr{E}_a^j = \mathscr{P}_a^j = 0$

for a **static** body "a" , $H_a^{kj} = 0$

$\Big\} \Rightarrow a_a^j\big|_{self} = 0$

Vector integrals ($c=G=1$):

$$t_a^j \equiv \int_a \frac{\rho^*\rho'^*\,v'^2\,(\vec x - \vec x')^j}{|\vec x - \vec x'|^3}\,d^3x\,d^3x'$$

$$\mathscr{T}_a^j \equiv \int_a \frac{\rho^*\rho'^*\,v'^j\,\vec v'\cdot(\vec x - \vec x')}{|\vec x - \vec x'|^3}\,d^3x\,d^3x'$$

$$\mathscr{T}_a^{**j} \equiv \int_a \frac{\rho^*\rho'^*\,[\vec v'\cdot(\vec x - \vec x')]^2\,(\vec x - \vec x')^j}{|\vec x - \vec x'|^5}\,d^3x\,d^3x'$$

$$\Omega_a^j \equiv \int_a \frac{\rho^*\rho'^*\rho''^*\,(\vec x - \vec x')^j}{|\vec x' - \vec x''|\,|\vec x - \vec x'|^3}\,d^3x\,d^3x'\,d^3x''$$

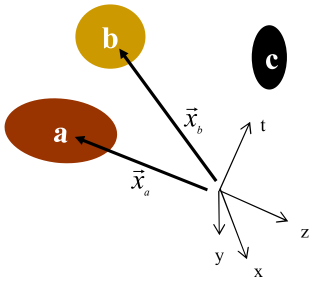
$$\mathscr{E}_a^j \equiv \int_a \frac{\rho^*\rho'^*\,\Pi'\,(\vec x - \vec x')^j}{|\vec x - \vec x'|^3}\,d^3x\,d^3x'$$

$$\mathscr{P}_a^j \equiv \int \frac{\rho^*\,p'\,(\vec x - \vec x')^j}{|\vec x - \vec x'|^3}\,d^3x\,d^3x'$$

Tensor integral ($c=G=1$):

$$H_a^{kj} \equiv \int_a \frac{\rho^*\rho'^*\,v'^k\,(\vec x - \vec x')^j}{|\vec x - \vec x'|^3}\,d^3x\,d^3x'$$

b) **Quasi-Newtonian acceleration** of the center of mass of body "*a*":



$$\vec{a}_a\big|_{Newton} = m_a^{-1}\,(m_P)_a^{jk}\,\mathfrak{U}_{,k}$$

total mass — passive mass tensor — Derivative of quasi-Newtonian potential

with $\mathfrak{U}(\vec{x}_a) = \sum_{b\neq a} \frac{m_A(\hat{n}_{ab})\big|_b}{r_{ab}}$ where $\begin{cases} \hat{n}_{ab} \equiv \frac{\vec{x}_{ab}}{r_{ab}} \\ r_{ab} \equiv |\vec{x}_{ab}| \\ \vec{x}_{ab} \equiv \vec{x}_a - \vec{x}_b \end{cases}$

$(m_P)_a^{jk}$ given in [Will 1993, p150]

…or, more convenient with a mass independent from direction:

$$\tilde{m}_I^{jk}\big|_a\; a_a^k\big|_{Newton} = \tilde{m}_P^{jk}\big|_a\; \mathfrak{U}_{,k} \quad \text{with} \quad \mathfrak{U}^{lm} = \sum_{b\neq a} \frac{\tilde{m}_A^{mq}\big|_b}{r_{ab}}\,\hat{n}_{ab}^q\,\hat{n}_{ab}^l$$

where

not present if only 2 bodies "*a*" and "*b*"

$\tilde{m}_P^{lm}\big|_a$ = **Inertial** mass tensor of body "a"
$\tilde{m}_A^{mq}\big|_b$ = **Active gravitational** mass tensor of body "b"
$\tilde{m}_I^{jk}\big|_a$ = **Passive gravitational** mass tensor of body "a"

functions of
- all PPN except α_3
- grav. self energy: $\Omega_{a(b)}$, $\Omega^{kj}{}_{a(b)}$
- P_b , E_b

[Will 1993, table 6.2, 151]

m_A = Active gravitational mass, the mass that determines the gravitational potential exhibited by a body. It is one type of m_{grav}.

m_P = Passive gravitational mass, the mass that determines the force on a body in a gravitational field. It is a second type of m_{grav}.

m_I = Inertial mass, the mass of the body to which the resulting acceleration (gravitational or non gravitational) is applied, in Newton's 1st law of motion.

Scalar integrals (*c*=*G*=*1*):

$$\Omega_a \equiv -\frac{1}{2}\int_a \frac{\rho^*\,\rho'^*}{|\vec{x}' - \vec{x}|}\,d^3x\,d^3x' \qquad \text{gravitational self energy}$$

$$P_a \equiv \int_a p\,d^3x$$

$$E_a \equiv \int_a \rho^*\,\Pi\,d^3x$$

Tensor integral (*c*=*G*=*1*):

$$\Omega_a^{ij} \equiv -\frac{1}{2}\int_a \frac{\rho^*\,\rho'^*\,(\vec{x}-\vec{x}')^i\,(\vec{x}-\vec{x}')^j}{|\vec{x}-\vec{x}'|^3}\,d^3x\,d^3x' \qquad \text{gravitational self energy}$$

In the most general case:

$$\left.\tilde{m}_P^{\ lm}\right|_a \neq \left.\tilde{m}_I^{\ jk}\right|_a$$ and are functions of gravitational self energy

Passive gravitational mass tensor of body "a"

Inertial mass tensor of body "a"

$\equiv$ **NORDTVEDT effect**

➡ ~~GWEP~~ but WEP, EEP, Eötvös experiments are ok,

Because: negligible self energy for {test particles, lab masses} : $\frac{\Omega_a}{m_a}, \frac{\Omega_a^{\ ij}}{m_a}$ <<< 1

GWEP = Gravitational Equivalence Principle.
Applies to any body, test-particles or extended bodies.

WEP = Weak Equivalence Principle.
Applies only to test-particles (laboratory masses).

Particular cases:

in **fully-conservative theories**, $\alpha_3, \zeta_1, \zeta_2, \zeta_3, \zeta_4 = 0$

⇩

$$\left\{ \begin{array}{l} \tilde{m}_A{}^{jk}\big|_a = \tilde{m}_P{}^{jk}\big|_a = \tilde{m}_I{}^{jk}\big|_a = \text{function of } \left(m_a; \beta, \gamma, \xi; \Omega_a, \Omega_a{}^{jk}\right) \\ \text{Quasi-Newtonian acceleration is antisymmetric under } a \leftrightarrow b\text{: action=reaction} \end{array} \right\}$$

in **General Relativity** or **Newtonian gravitation** ⇨ mass tensors are isotropic

$$\tilde{m}_A\big|_a = \tilde{m}_P\big|_a = \tilde{m}_I\big|_a = m_a$$

for **spherically symmetric bodies**

$$\frac{m_P|_a}{m_a} = 1 + \left(4\beta - \gamma - 3 - \frac{10}{3}\xi - \alpha_1 + \frac{2}{3}\alpha_2 - \frac{2}{3}\zeta_1 - \frac{\zeta_2}{3}\right)\frac{\Omega_a}{m_a}$$

$\equiv \eta$ **NORDTVEDT parameter**

$$\frac{m_A|_b}{m_b} = 1 + \left(4\beta - \gamma - 3 - \frac{10}{3}\xi - \frac{1}{2}\alpha_3 - \frac{1}{3}\zeta_1 - 2\zeta_2\right)\frac{\Omega_b}{m_b} + \zeta_3\frac{E_b}{m_b} - \left(\frac{3}{2}\alpha_3 + \zeta_1 - 3\zeta_4\right)\frac{P_b}{m_b}$$

ex: $\eta_{GR} = 0$

$$\eta_{TS} = \frac{1}{2+\omega_0} + 4\Lambda_0$$

Conservation of momentum:

In fully-conservative theories: action = reaction and thus $\sum_a m_a \cdot a_a^i\big|_{Newt} = 0$

For spherically symmetric bodies:

$\Omega_a{}^{kj} \approx \frac{1}{3}\delta^{kj}\Omega_a$ and passive gravitational mass tensor is combined with inertial mass tensor into m_P

c) n-body accelerations of center of mass of body "*a*"
due to PN corrections
in a point-mass geodesic derivation (∍classical PPN):

$$\vec{a}_a\Big|_{n\,body} = +\sum_{b\neq a}\frac{m_b x^j_{ab}}{r_{ab}}\left\{\begin{aligned}&+(2\gamma+2\beta)\frac{m_b}{r_{ab}}+\left(2\gamma+2\beta+1+\frac{1}{2}\alpha_1-\zeta_2\right)\frac{m_a}{r_{ab}}\\&+(2\beta-1-2\xi-\zeta_2)\sum_{c\neq a,b}\frac{m_c}{r_{bc}}+(2\gamma+2\beta-2\xi)\sum_{c\neq a,b}\frac{m_c}{r_{ac}}\\&-1/2\cdot(1+2\xi+\alpha_2-\zeta_1)\sum_{c\neq a,b}\frac{m_c\,\vec{x}_{ab}\cdot\vec{x}_{bc}}{r^3_{bc}}-\xi\sum_{c\neq a,b}\frac{m_c\,\vec{x}_{bc}\cdot\vec{x}_{ac}}{r^3_{ac}}\\&-\gamma v_a^2+1/2\cdot(4\gamma+4+\alpha_1)\vec{v}_a\cdot\vec{v}_b-1/2\cdot(2\gamma+2+\alpha_2+\alpha_3)v_b^2\\&+1/2\cdot(\alpha_1-\alpha_2-\alpha_3)w^2+1/2\cdot\alpha_1\vec{w}\cdot\vec{v}_a+1/2\cdot(\alpha_1-2\alpha_2-2\alpha_3)\vec{w}\cdot\vec{v}_b\\&+3/2\cdot(1+\alpha_2)(\vec{v}_b\cdot\hat{n}_{ab})^2+3/2\cdot\alpha_2(\vec{w}\cdot\hat{n}_{ab})^2+3\alpha_2(\vec{w}\cdot\hat{n}_{ab})(\vec{v}_b\cdot\hat{n}_{ab})\end{aligned}\right\}$$

$$-1/2\cdot(4\gamma+3-2\xi+\alpha_1-\alpha_2+\zeta_1)\sum_{b\neq a}\frac{m_b}{r_{ab}}\sum_{c\neq a,b}\frac{m_c x^j_{bc}}{r^3_{bc}}$$
$$-\xi\sum_{b\neq a}\frac{m_b}{r^3_{ab}}(\delta_{jk}-3\hat{n}^j_{ab}\hat{n}^k_{ab})\sum_{c\neq a,b}m_c\left(\frac{x^k_{ac}}{r_{ac}}-\frac{x^k_{bc}}{r_{bc}}\right)$$

not present if only 2 bodies "*a*" and "*b*"

$$+\sum_{b\neq a}\frac{m_b}{r^3_{ab}}\vec{x}_{ab}\left[(2\gamma+2)\vec{v}_a-(2\gamma+1)\vec{v}_b\right]v^j_a$$
$$-1/2\cdot\sum_{b\neq a}\frac{m_b}{r^3_{ab}}\vec{x}_{ab}\left[(4\gamma+4+\alpha_1)\vec{v}_a-(4\gamma+2+\alpha_1-2\alpha_2)\vec{v}_b+2\alpha_2\vec{w}\right]v^j_b$$
$$-1/2\cdot\sum_{b\neq a}\frac{m_b}{r^3_{ab}}\vec{x}_{ab}\left[\alpha_1\vec{v}_a-(\alpha_1-2\alpha_2)\vec{v}_b+2\alpha_2\vec{w}\right]w^j$$

2-*body system*, acceleration on body "*a*":

$$\vec{a}_a=\vec{a}_a\Big|_{self}+\vec{a}_a\Big|_{Newton}+\vec{a}_a\Big|_{n\,body}$$

$$a^j_a\Big|_{self}=-m_a^{-1}\left[\begin{aligned}&+\frac{1}{2}(\alpha_3+\zeta_1)t^j_a+\zeta_1\left(\mathscr{T}^j_a-\frac{3}{2}\mathscr{T}^{**j}_a\right)\\&+\zeta_2\Omega^j_a+\zeta_3\mathscr{E}^j_a+3\zeta_4\mathscr{P}^j_a\\&-\alpha_3(w+v_a)^k H^{kj}_a\end{aligned}\right]$$

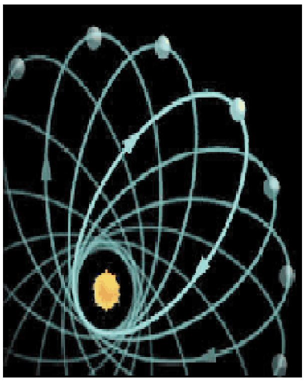
$$\tilde{m}^{jk}_I\Big|_a\; a^k_a\Big|_{Newton}=\tilde{m}^{jk}_P\Big|_a\;\mathfrak{U}_{,k}\qquad\text{with}\qquad\mathfrak{U}^{lm}=\frac{\tilde{m}^{mq}_A\Big|_b}{r_{ab}}\;\hat{n}^q_{ab}\;\hat{n}^l_{ab}$$

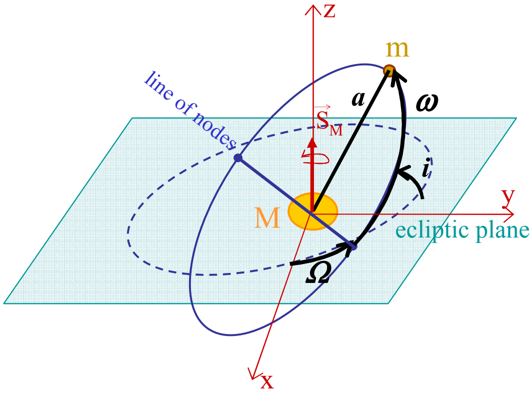
$$\vec{a}_a\Big|_{n\,body}=+\frac{m_b x^j_{ab}}{r_{ab}}\left\{\begin{aligned}&+(2\gamma+2\beta)\frac{m_b}{r_{ab}}+\left(2\gamma+2\beta+1+\frac{1}{2}\alpha_1-\zeta_2\right)\frac{m_a}{r_{ab}}\\&-\gamma v_a^2+1/2\cdot(4\gamma+4+\alpha_1)\vec{v}_a\cdot\vec{v}_b-1/2\cdot(2\gamma+2+\alpha_2+\alpha_3)v_b^2\\&+1/2\cdot(\alpha_1-\alpha_2-\alpha_3)w^2+1/2\cdot\alpha_1\vec{w}\cdot\vec{v}_a+1/2\cdot(\alpha_1-2\alpha_2-2\alpha_3)\vec{w}\cdot\vec{v}_b\\&+3/2\cdot(1+\alpha_2)(\vec{v}_b\cdot\hat{n}_{ab})^2+3/2\cdot\alpha_2(\vec{w}\cdot\hat{n}_{ab})^2+3\alpha_2(\vec{w}\cdot\hat{n}_{ab})(\vec{v}_b\cdot\hat{n}_{ab})\end{aligned}\right\}$$
$$+\frac{m_b}{r^3_{ab}}\vec{x}_{ab}\left[(2\gamma+2)\vec{v}_a-(2\gamma+1)\vec{v}_b\right]v^j_a$$
$$-1/2\cdot\frac{m_b}{r^3_{ab}}\vec{x}_{ab}\left[(4\gamma+4+\alpha_1)\vec{v}_a-(4\gamma+2+\alpha_1-2\alpha_2)\vec{v}_b+2\alpha_2\vec{w}\right]v^j_b$$
$$-1/2\cdot\frac{m_b}{r^3_{ab}}\vec{x}_{ab}\left[\alpha_1\vec{v}_a-(\alpha_1-2\alpha_2)\vec{v}_b+2\alpha_2\vec{w}\right]w^j$$

B2. Main relativistic effects in celestial mechanics and tests of relativistic theories of gravitation

- **Perihelion advance of planets ($\Delta\omega$):**

Orbit parameters: i = inclination
ω = pericenter
Ω = node





a) History

- **1859:** Le Verrier's observations : deviation of Mercury's orbit from Newtonian's predictions ($\Delta\omega_{obs}$), not due to known planets Vulcain??
- **1877:** No vulcain seen!!!
- **1885:** Newcomb's attempt: account for $\Delta\omega_{obs}$ with modified gravitational field (solar quadrupole moment $J_{2\odot}$)
  - ➡ 1st time that $J_{2\odot}$ is associated with $\Delta\omega$
  - ➡ corresponding oblateness of 500 arcms ruled out by solar observations
- **1915:** Einstein's GR accounts for $\Delta\omega_{obs}$
- **1966-1974:** Dicke et al. measure, through visual oblateness, $J_{2\odot} \sim 10^{-5}$
  - ➡ $\Delta\omega_{GR}$ in trouble, $\Delta\omega_{BD}$ with $\omega_{BD} \approx 5$ ok??!
  - ➡ controversies about oblateness measurements!
- **Present:**
  - Einstein's GR accounts for almost all $\Delta\omega_{obs}$; and $J_{2\odot} \sim 10^{-7}$
  - $\Delta\omega_{obs}$ is a cornerstone test of GR ... or alternative theories of gravitation!
  - Mercury = interesting lab: inner most of the terrestrial planets, relativistic motion, close approach of the Sun
  - $J_{2\odot}$ (<<< Newcomb's) can not be discarded

GR = General Relativity theory

BD = Brans-Dicke theory

Note that to solve the conflict theory-observations, a ring of planetoids or a deviation from the inverse square law of gravitation where also proposed.

$$J_{2\odot} = \frac{(C-A)}{M_\odot R_\odot^2}$$ with A, C, the moments of inertia about the body's rotation and equatorial axes respectively

b) Theoretical calculation

$$\Delta\omega = \Delta\omega_{0\,GR} \bullet \left[+\frac{1}{3}(2\alpha^2 + 2\alpha\gamma - \beta) \;\; \frac{1}{6}(2\alpha_1 - \alpha_2 + \alpha_3 + 2\zeta_2)\frac{M_\odot\, m}{(M_\odot + m)^2} \;\; - \frac{R_\odot^2}{R_{Schw\odot}\;\alpha\, a\,(1\text{-}e^2)} J_{2\odot}(3\sin^2 i - 1) \right]$$

neglect

- $+\frac{1}{3}(2\alpha^2 + 2\alpha\gamma - \beta)$ → ~ 10^{-4} owing to PPN constraints on β (from η) and γ
- $\frac{M_\odot\, m}{(M_\odot + m)^2}$ → ~ 2×10^{-7} for *m* = Mercury
- $(2\alpha_1 - \alpha_2 + \alpha_3 + 2\zeta_2)$ → = 0 if fully conservative theory
- $J_{2\odot}(3\sin^2 i - 1)$ term → ~ $3 \times 10^{-4} \cdot (J_{2\odot}/10^{-7})$

with $(a, e, i) \equiv$ orbital parameters of *m*

$R_{schw\odot} \equiv 2\frac{G\,M_\odot}{c^2}$ Schwarzschild radius of Sun

$J_{2\odot} \equiv$ gravitational quadrupole moment of Sun

$R_\odot \equiv$ mean radius of the Sun

$$\Delta\omega_{0\,GR} \equiv \frac{6\pi\, G\,(M_\odot + m)}{a\,(1\text{-}e^2)\,c^2}$$

Planets	a [AU]	e [/]	i [°]	$\Delta\omega_{0\,GR}$ [arcseconds/ revolution]	Revolutions / Century	$\Delta\omega_{GR}$ [arcseconds/ century]
Mercury	0.3870989	0.2056	7.005	0.1035 ”	415.2	43.01
Venus	0.7233320	0.0068	3.395	0.053 ”	163	8.6
Earth	1.0000001	0.0167	0.0001	0.038 ”	100.0	3.8
Icarus	1.077936	0.827	22.855	0.113 ”	89.3	10.1

Values for the relativistic advances were computed with

- *G, c, AU_to_meter, mean sideral day, sideral year* taken from http://ssd.jpl.nasa.gov/astro_constants.html

- $M_{\odot}$, $R_{\odot}$ taken from http://nssdc.gsfc.nasa.gov/planetary/planetfact.html

- http://nssdc.gsfc.nasa.gov/planetary/planetfact.html for planetary parameters (*a, e, i*) ;
http://ssd.jpl.nasa.gov/phys_props_planets.html for orbital period;
http://ssd.jpl.nasa.gov/cgi-bin/et for minor planet Icarus

c) Difficulty of observations: example of Mercury

- Not directly observable: other effects contribute to perihelion advance …

$$\Delta\omega = \Delta\omega_{0\,GR} \bullet \left[+\frac{1}{3}(2\alpha^2 + 2\alpha\gamma - \beta) - \frac{R_\odot^2}{R_{Schw\odot}\,\alpha\, a\,(1-e^2)} J_{2\odot} (3\sin^2 i - 1) \right]$$

$\Delta\omega_{0\,GR}$: 42,98 arcsec/century

Perihelion advance of Mercury [arcsec/century]	
PPN	~ 43
$J_{2\odot}$	
equinoxes	~ 5000
planets:	
Venus	~ 280
Jupiter	~ 150
Autres	~ 100

➡ $\Delta\omega_{obs}$ obtained by comparing 2 fits of the osculating elements of the orbits: {Newtonian model; Relativistic model}

- purely relativistic PPN and $J_{2\odot}$ contributions are still strongly correlated

d) Existing estimates of $J_{2\odot}$?

- ≠methods give ≠estimates

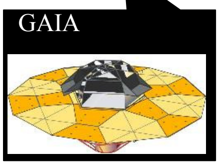
Stellar structure equations
+
Differential rotation model
ex: [Godier, Rozelot 1999] ~ 1,6 10^{-7}

Theory of the solar figure
ex: [Rozelot, Lefebvre 2003] ~ 6,5 10^{-7}

Helioseismology
ex: [Pijpers 1998] ~ 2,2 10^{-7}

- Dynamical consequences: ephemerides, light deflection
- Constraints on solar models

➡ **Dynamical** estimation of $J_{2\odot}$, independent from solar models



Problems connected with solar J2 estimations:

-Today, the scientific community agrees on the order of magnitude of J2: 10**(-7).
However, precise estimates still strongly depend on the method used: stellar equations combined with a differential rotation model, the Theory of Figures applied to the Sun, or inversions techniques applied to helioseismology.

- the problem is to have a precise value of J2 to adopt by the scientific community for dynamical contributions. Example: beta PPN parameter is strongly correlated to J2 in ephemerides (50-80 % according to observation sets); or to compute light deflection in the neighborhood of the Sun.

- a precise estimate of J2 is crucial to constrain solar models.

Values of J2 [orange terms] are based not only on (difficult) solar observations (helioseismology, solar diameter), and also on solar (density and rotation) models. Hence we need to have a dynamical estimate of J2, independent from solar models… [Pireaux et al 2003]

e) Example of a dynamical estimate of $J_{2\odot}$

- **Direct influence** of $J_{2\odot}$ on the orbital motion of planets:
  - perihelion advance
  - planets spins
  - variation of ecliptic plane

- ∃ spin-orbit coupling of Solar System bodies

  ex: importance of coupling in ⊕ - ☾ system

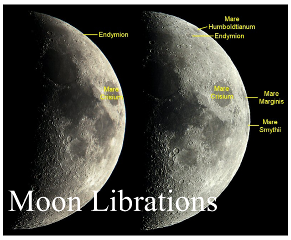
  propagates the influence of $J_{2\odot}$ ≡ **indirect influence**

  ex: $J_{2\odot}$ → ⊕ → ☾

➡ **dynamical** constraints on $J_{2\odot}$

ex: by librations ☾ : $J_{2\odot} \leq 3 \cdot 10^{-6}$

[Rozelot, Rösch]
[Bois, Girard]

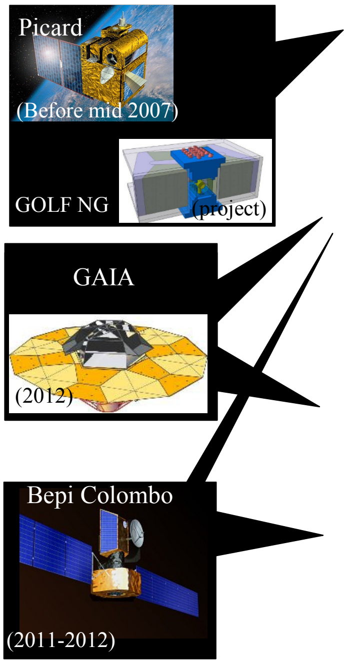


Ecliptic plane variations induced by solar J2 are weaker than those induced by planets but they act retroactively on planetary orbits and can influence solar system long-term stability.

See [Rozelot et al. 1997], [Bois et al. 1999] for a demonstration by Rozelot and Rösch, Bois and Girard of the importance of couplings in Moon-Earth, and inferred dynamical constraint on solar J2.

Moon rotation (librations) is well measured, at the milliarcsecond thanks to Lunar Laser Ranging.

f) Futur observational developments



- Better measurements of the solar diameter : space missions
- Better estimate of PPN
- Possible decorrelation $J_{2\odot} \longleftrightarrow$ PPN in $\Delta\varpi$
- Measurement of the precession of the orbital plane around the polar axis of the Sun : $J_{2\odot}$
- …

Space missions like PiCARD or GOLF NG (Global Oscillation at Low Frequencies, Nouvelle Génération): better measurement of the solar diameter at different latitudes because no Earth atmosphere effect (precision of about 10**(-8) on solar J2).

GAIA or BepiColombo: better estimates of PPN parameter gamma.

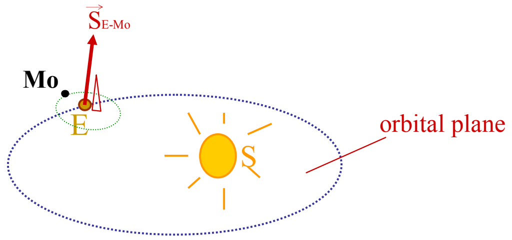
Mercury orbiter (BepiColombo mission): precise measurement of the position of Mercury allowing to « measure » Mercury perihelion advance; better constraints on the Nordtvedt effect [precision of 2 10**(-5)]. Combining those with constraints on gamma would allow to access to solar J2.

GAIA: decorrelate solar J2 from PPN in perihelion advance, thanks to the different dependency in « a » and « e », with a sampling on minor planets.

BepiColombo: should measure the precession of line of node of Mercury (not depending on PPN), hence give a dynamical estimate of solar J2.
The whole BepiColombo mission should estimate solar J2 with a precision of 2 10**(-9).

- **Geodetic precession of Earth-Moon system:**

E-Mo system analogous to a gyroscope in the gravitational field of S



In curved space-time, ∃ precession of the gyroscope spin axis ($\vec{S}$)

➡ $$\frac{d\vec{S}_{E-Mo}}{d\tau} = \vec{\Omega}_{GP\ E\text{-}Mo} \times \vec{S}_{E-Mo} \quad \text{with} \quad \vec{\Omega}_{GP\ E\text{-}Mo} = -\frac{3}{2}\frac{1}{c^2}\vec{v}_{E-Mo} \times \vec{\nabla} w_{\text{ext}\ E-Mo}(\vec{x}_n)$$

$\frac{3}{2} \rightarrow -\left(\frac{1}{2}+\gamma\right)$

$$w_{\text{ext E-Mo}} \approx U_S$$

➡ **1916:** 1st calculated by De Sitter: ~ 2 arcsec/century
1988: 1st detected with lunar ranging + radio interferometry data

➡ measured to ~ 0.7% with LLR [Dickey et al. 1994], [Williams et al. 1996]

➡ = test of PPN parameter γ

$w_{\text{ext}\,n}$ = external gravitational potential to which body n is submitted

U_S = gravitational Newtonian potential due to the Sun

• Nordtvedt Effect:

a) In general:

Spherical body M

$$\vec{F} = m_I \vec{a} \quad \text{with} \quad \vec{F} = -G \frac{m_P M_A}{|r|^2} \vec{r} \equiv m_P \vec{g}$$

$$\frac{m_P}{m_I} = 1 - \eta \cdot \frac{E_{grav}}{m_I c^2} \quad \text{with} \quad E_{grav} \equiv \frac{\int \rho(x) \cdot \rho(x') \frac{d^3x \, d^3x'}{|\vec{x} - \vec{x}'|}}{2 \int \rho(x) \, d^3x}$$

$\Rightarrow \vec{a}(E_{grav})$

ex: uniform density sphere (m,R) $E_{grav} = -\frac{3}{5} \frac{G m^2}{R}$

$E_{grav} / m_{iner} c^2$	Lab size objects	Moon	Earth	Jupiter	Sun
	$<10^{-27}$	0.2×10^{-10}	4.6×10^{-10}	10^{-8}	10^{-5}

Presently non observable

$$\eta \equiv 4\beta - \gamma - 3 - 10\xi/3 - \alpha_1 + 2\alpha_2/3 - 2\zeta_1/3 - \zeta_2/3$$

ex: $\eta_{GR} = 0, \quad \eta_{TS} = \frac{1}{2 + \omega} + 4\Lambda$

m_{grav} = gravitational mass

m_P = Passive gravitational mass, the mass that determines the force on a body in a gravitational field. It is a second type of .

m_I = inertial mass

E_{grav} = - gravitational binding energy

U = gravitational external potential

$\rho(x)$ = matter density of the body subject to the gravitational potential

η = Nordtvedt parameter

b) Application to Earth-Moon system:

$M = M_{Sun}$ gravitational potential source

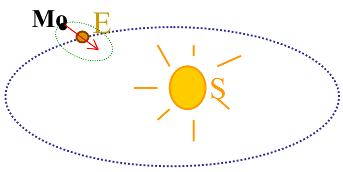
$m = m_E$ or m_{Mo} attracted mass

➡ $a_{E \text{ due to } S} \neq a_{Mo \text{ due to } S}$

➡ polarization of the Earth-Moon orbit:

$$\delta r_{E-Mo} = 13.1 \cdot \eta \cdot \cos(\varpi_0 - \varpi_s) \cdot t \quad [\mathrm{m}]$$ directed towards S as seen from E

with (ϖ_0, ϖ_s) angular frequencies of orbits of (S,Mo) around E



➡ Lunar Laser Ranging Experiment (LURE): **no evidence of Nordtvedt effect**

-1969: 1st successful acquisition with Apollo11 Moon reflector

- since 1969: measurements from a network of observatories, accuracy ≈50ps (1cm), least-square fit to theoretical model of Mo motion and traveling signal (∋ perturbations of S and planets, tidal interactions, Mo librations, E orientation, atmosphere and signal propagation, observatory location and PPN parameters)

Mo = Moon

E = Earth

S = Sun

The Earth-Moon system measurements lead to a test of the Nordtvedt effect, because they are massive bodies with non-negligible gravitational energy. But the WEP effect has to be substracted using WEP constraints from laboratory experiments, because the two planets differ in composition and hence should be affected by WEP violations.

• Violation of the conservation of momentum:

Present model of the Moon structure

If ∃violation of Newton 3rd law ➡ momentum non-conserving self-force

➡ secular acceleration of Lunar orbit

Info on Mo orbit from LLR + on tidal effects on E-Mo system from satellites

➡ constraints on anomalous secular acceleration

➡ $\left| \left(\left.\frac{m_A}{m_P}\right|_{Al} - \left.\frac{m_A}{m_P}\right|_{Fe} \right) \Big/ \left.\frac{m_A}{m_P}\right|_{Fe} \right| < 4\ 10^{-12}$

Contribution of E_{elec} in PPN: $\frac{m_A}{m_P} = 1 + \frac{1}{2}\zeta_3 \cdot \frac{E_{elec}}{m_P c^2}$ [Will 1993, p214-215]

➡ $|\zeta_3| < 1\ 10^{-8}$ [Barlett et al. 1986]

E_{elec} = Electrostatic binding energy of an atomic nucleus of a given type

Newton's 3rd law = 'action is equal to reaction'.
In other words, the active gravitational mass should equal the passive gravitational mass.
This guaranties the equality of action and reaction and the conservation of momentum, at least in the Newtonian limit:

$\vec{F}_{mM} = m_I \vec{a}_m$ with $\vec{F}_{mM} = -G\frac{m_P M_A}{|r_{Mm}|^2}\vec{r}_{Mm}$ …on m due to M

$\vec{F}_{Mm} = M_I \vec{a}_M$ with $\vec{F}_{Mm} = -G\frac{M_P m_A}{|r_{mM}|^2}\vec{r}_{mM}$ …on M due to m

if $\left(\frac{m_A}{m_P} - \frac{M_A}{M_P}\right) \Big/ \frac{M_A}{M_P} \neq 0$ then $\vec{F}_{Mm} \neq -\vec{F}_{mM}$

hence net self-force on the center of mass of (m, M)

$$F = \left(\frac{m_A}{m_P} - \frac{M_A}{M_P}\right) G m_P M_P \Big/ r_{mM}^{\ 2} = 0$$

m_A = Active gravitational Mass, the mass that determines the gravitational potential exhibited by a body

m_P = Passive gravitational mass, the mass that determines the force on a body in a gravitational field.

In [Barlett et al. 1986], *G* was assumed to be constant. If not, an independent experiment is needed to assign the anomaly to a difference in passive-active masses or to *G* variation.

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