Centre de Compétence Technique
CCT2-Mécanique Orbitale

Relativity and time transformations: an operational point of view

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Part. 1:

TIME TRANSFORMATIONS

Some comments and precisions in the vocabulary are needed, since for exemple the monograph 2000 by Moyer [Moyer 2000] was published in 2000 before the IAU2000 resolutions. This monograph is widely used in orbitography, but the terminoly is not always correct from a relativistic point of vue. For exemple, proper time is assumed to be the same as TAI… where as the proper time of a clock refers to WHAT the clock really measures, atomic scale is rather which units (HOW) are used by the clock.

Plan

➡ **Part. 1: TIME TRANSFORMATIONS**

1. MOTIVATION: time and frequency measurement in the Solar System
   a) Precision of clocks
   b) Location of clocks
   c) Observational errors
   d) Rapid order of magnitude of relativistic effects

2. PROPER AND COORDINATE TIMES

3. REFERENCE SYSTEMS

4. ASTRONOMICAL TIME SCALES (Earth rotation)

5. RELATIVISTIC TIME SCALES (atomic clocks)
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6. EXACT and APPROXIMATE TRANSFORMATIONS
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   b) in BCRS, in terms of TCB scale
   c) in BCRS, in terms of TDB scale
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   e) Computing the Relativistic Time Dilation Integral

1. MOTIVATION: time and frequency measurement in the Solar System

a) Precision of clocks:

		Accuracy in ν	Stability of ν $\sigma_y(\tau)$ (s)	Reference
Present best	Cs-fountain clocks	< 2 E-15	4 E-14 $\tau^{-1/2}$	[Lemonde et al 2001] [Weyers et al 2001]
Future best	Laser cooled RB clocks Cs space borne clocks	few 1 E-17	1 E-14 $\tau^{-1/2}$	[Bize et al 1999] [Lemonde et al 2001]

b) Location of clocks:

BCRS: SORT project: clocks within ($|\vec{X}|$) 0.25 AU from the Sun

GCRS: clocks on the geoid up to geostationary orbits ($|\vec{x}|$ < 50 000 km)

c) Observational errors:

RANGING: DSN spacecraft ~ 1m ➡ error relative to Pluto's distance ~ 2 E-13 (s) ≡ **time derivative error**

PULSAR TIMING: on daily mean pulse arrival epochs of Pulsar ≤ 0.1 E-6 (s) ≡ **error on time**

➡ **Need a COHERENT conventional model for: time coordinates + time transformations**

« Present » precision GR existing astrometric quantities (planetary ephemerides)

The uncertainties allowed in the time transformations are < 5 E-18 in rate and < 0.2 E-12 s in amplitude for quasi periodic terms.

The spatial domain of validity of transformation has also to verify the present (future) observations available.

SORT = Solar Orbit Relativity Test.

$\sigma_y(\tau)$ (s) = Allan deviation. It provides a measure of the rms (residual mean squares) fractional frequency deviation among clocks, due to intrinsic noise processes in the clocks. Additional systematic effects (like frequency drifts or offsets must be accounted separately).

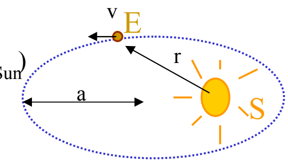
d) A rapid order of magnitude of relativistic effects:

In barycentric coordinates ...

Let - an horloge (τ) be located at **the center of the Earth**

- **Barycentric** Coordinate Reference System (t = TCB)

- assume ~ only the **Sun** is present (U(r) = GM/r with M=M $_{Sun}$)
  ~ keplerian orbit around Sun (r, a, v, M, e, n)



From the Schwarzschild line element:

$$\frac{d\tau}{dt} = \sqrt{\left[1-2\frac{U}{c^2}\right]-\left[1+2\frac{U}{c^2}\right]\cdot\frac{v^2}{c^2}} \approx 1-\frac{1}{2}\frac{v^2}{c^2}-\frac{U}{c^2}$$

$$\Rightarrow \quad \tau \approx t - \frac{1}{c^2}\int\left[U+\frac{1}{2}v^2\right]dt$$

$$\approx \left(1-\frac{3}{2}\frac{GM}{c^2 a}\right)\cdot t \; - \; 2\frac{e}{n}\frac{GM}{c^2 a}\cdot\sin M \; - \; \frac{e^2}{n}\frac{GM}{c^2 a}\cdot\sin 2M \; + \; \Theta(e^3)$$

Shift in frequency ~1.49 E-8 | 1st periodic term ~1.65 ms | 2nd periodic term ~14 µs

Keplerian orbits:

ω= argument du périgée [/]

Ω= argument du noeud [/]

a= demi-grand axe [m]

e= excentricité [/]

M= anomalie moyenne [°]

n= mouvement moyen [°/jours] ou [°/sec]

E= anomalie d'excentricité [/]

r= rayon de l'orbite [m]

i= inclinaison [°]

$$\left(\frac{dr}{dt}\right)^2 = GM\left(\frac{2}{r}-\frac{1}{a}\right)$$

$$\frac{dE}{dt} = \frac{1}{r}\sqrt{\frac{GM}{a}}$$

$$\frac{1}{r} = \frac{1}{a} + \frac{e}{r}\cos E$$

if circular orbit: $\frac{d\vec{r}}{dt}\cdot\vec{r} = \sqrt{GM\,a}\,e\sin E$

$$M = n\cdot t$$

$$M = (n+\omega)\cdot t$$

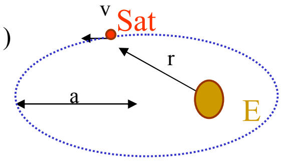
In geocentric coordinates ...

Let - an horloge (τ) be located on an **Earth satellite**

- **Geocentric** Coordinate Reference System (t = TCG)

- assume ~ only the **Earth** is present (U(r) = GM/r with $M=M_{Earth}$)

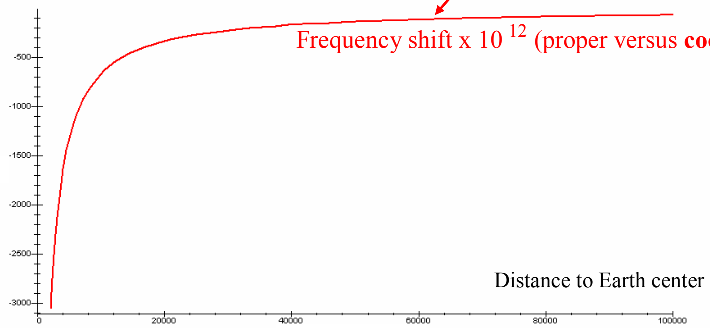
  ~ keplerian orbit around Earth (r, a, v, M, e, n)



From the Schwarzschild line element:

$$\frac{d\tau}{dt} = \sqrt{\left[1-2\frac{U}{c^2}\right]-\left[1+2\frac{U}{c^2}\right]\cdot\frac{v^2}{c^2}} \approx 1-\frac{1}{2}\frac{v^2}{c^2}-\frac{U}{c^2} \quad = 1-\frac{1}{2c^2}\left(\frac{2GM}{r}-\frac{GM}{a}\right)-\frac{U}{c^2}$$

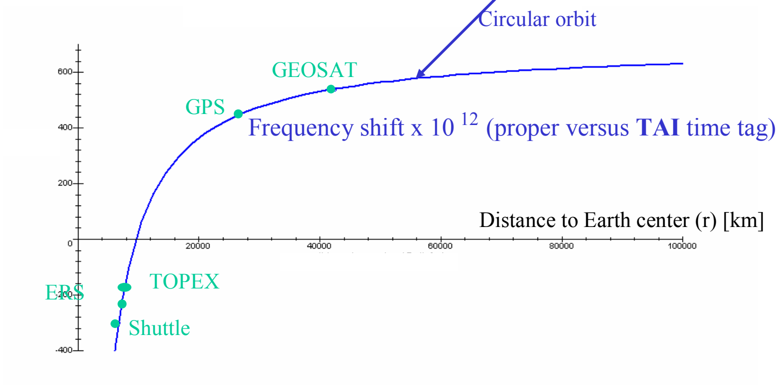
Circular orbit



With respect to an atomic clock on the rotating geoid (TAI),

Feeling the effective potential $\Phi_0 \equiv \frac{\mathrm{GM}}{R_{E\,\mathrm{equateur}}} + \frac{\mathrm{GM}\cdot \mathrm{J2}}{2R_{E\,\mathrm{equateur}}} + \frac{\omega_E^2 \cdot R_{E\,\mathrm{equateur}}^2}{2}$

$$\Rightarrow \quad \frac{d\tau}{dTAI} \approx 1 - \frac{1}{2}\frac{v^2}{c^2} - \frac{(U-\Phi_0)}{c^2} = 1 - \frac{1}{2c^2}\left(\frac{2GM}{r} - \frac{GM}{a}\right) - \frac{(U-\Phi_0)}{c^2}$$



A slowing down of the clocks on boards of satellites…

Altitude in km of the satellites (to get the distance from the Earth center, add the equatorial radius)

r_km_shuttle = 300

r_km_ERS = 800

r_km_TOPEX = 1300

r_km_LAGEOS = 7000

r_km_GPS = 20 000

r_km_GEOSAT = 36 000

computation of the corresponding (dtau/dTAI-1)*10**12

shift_shuttle = -299.238

shift_ERS = -229.849

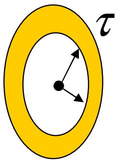
shift_TOPEX = -169.498

shift_GPS = 444.730

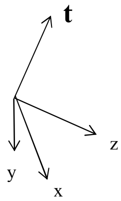
shift_GEOSAT = 539.948

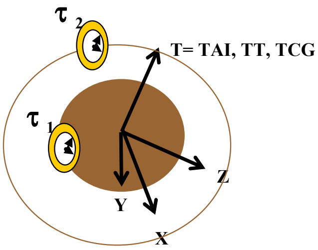
And the shift cancels at 9545 km

2. PROPER AND COORDINATE TIMES $\tau \leftrightarrow t$



« Proper time, τ, is an **observable** quantity; whereas coordinate time, t , is only a coordinate in the mathematical sense, which can only be **calculated** using τ. »





« Relativistic time dilation does not affect the ability of a clock to measure or maintain **its proper time**. But no physical clock can maintain coordinate time. »

« In relativity, **time is not absolute** »

We distinguish between:

- **coordinate time:** TCG, TCB

Usually written as T or t.

It is the time that appears in the metric as a coordinate, without any a forteriori approximation (unlike what Ashby does in his article about GPS (2003).

- **proper time:** τ

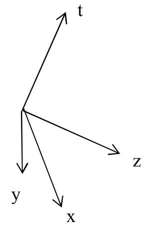
It is the time measured by a clock present where the measurement is made.

- **time tags:** TAI, TAIsat (i.e. TGPS, TPX)

Those are an average proper time (over orbital periodicity) which is estimated (thus dependant upon the order of the approximation used at the time the time tag was defined) for a clock located respectively at sea level or on a typical GPS or TOPEX satellite orbit.

3. REFERENCE SYSTEMS

BCRS

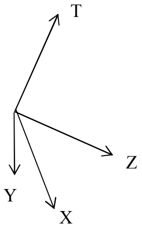


at order $\Theta\left(\frac{1}{c^5}\right)$

$$x^\mu = (ct, x, y, z)$$

$$g_{\mu\nu} = \begin{pmatrix} -1+2\frac{w(x,y,z)}{c^2}-2\left(\frac{w(x,y,z)}{c^2}\right)^2 & 0-4\frac{w^i(x,y,z)}{c^3} & 0-4\frac{w^i(x,y,z)}{c^3} & 0-4\frac{w^i(x,y,z)}{c^3} \\ 0-4\frac{w^i(x,y,z)}{c^3} & 1+2\frac{w(x,y,z)}{c^2} & 0 & 0 \\ 0-4\frac{w^i(x,y,z)}{c^3} & 0 & 1+2\frac{w(x,y,z)}{c^2} & 0 \\ 0-4\frac{w^i(x,y,z)}{c^3} & 0 & 0 & 1+2\frac{w(x,y,z)}{c^2} \end{pmatrix}$$

GCRS

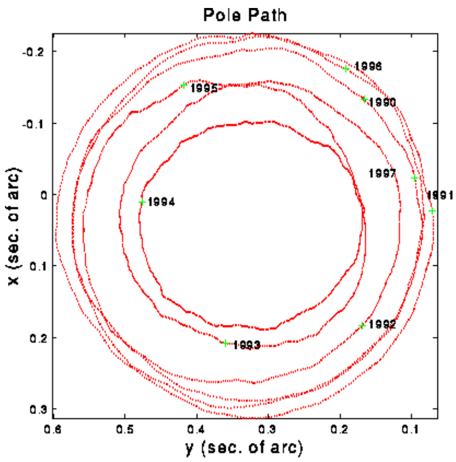


$$X^\mu = (cT, X, Y, Z)$$

$$G_{\mu\nu} = \begin{pmatrix} -1+2\frac{W(X,Y,Z)}{c^2}-2\left(\frac{W(X,Y,Z)}{c^2}\right)^2 & 0-4\frac{W^i(X,Y,Z)}{c^3} & 0-4\frac{W^i(X,Y,Z)}{c^3} & 0-4\frac{W^i(X,Y,Z)}{c^3} \\ 0-4\frac{W^i(X,Y,Z)}{c^3} & 1+2\frac{W(X,Y,Z)}{c^2} & 0 & 0 \\ 0-4\frac{W^i(X,Y,Z)}{c^3} & 0 & 1+2\frac{W(X,Y,Z)}{c^2} & 0 \\ 0-4\frac{W^i(X,Y,Z)}{c^3} & 0 & 0 & 1+2\frac{W(X,Y,Z)}{c^2} \end{pmatrix}$$

3. ASTRONOMICAL TIME SCALES:

connected with Earth rotation



[USNO 1996 web page]

… but the Earth rotation is not uniform (periodic changes and long term drifts)

➡ prefer to refer to atomic time scales!

UT = Universal Time. It is the time used for civilian live. It is connected to solar time, as it measures the Earth rotation with respect to the Sun. A ***solar year*** lasts 365.25 days. A version of it is **UT1 =** Earth rotation time, independent from observing location: it is corrected for the observer longitude with respect to Greenwich Meridian, and for observer's small shift in longitude due to polar motion. It is chosen so that a day of 24*60*60 s = 86400s UT1 is close to the duration of the mean solar day. The phase of UT1 is chosen so that the Sun crosses the Greenwich meridian at 12h UT1. UT1 contains 41 short period terms with periods between 5 and 35 days (=DUT1), caused long period solid Earth tides. **UTR** = Universal Time Regularized, it is obtained by substracting DUT1 from UT1. **UT0 =** Earth rotation time dependent from observing location (long, lat): it contains the effect of polar motion on the observed rotation of the Earth (polar motion -pole offsets (x,y) published by the IERS bulletins A/B- is equivalent to a change in latitude/longitude of points on the Earth 's surface with respect to the Earth instantaneous rotation axis).

$$UT0 = UT1 + \tan(lat) \cdot [x \cdot \sin(long) + y \cdot \cos(long)]$$

UTC = Coordinated Universal time. Broadcasted by WWV and other services, it is the standard time for 0° longitude. Since 1st January1992, it uses the SI second and is kept close to the UT time: it should not differ by more than 0.90 seconds from it. Beyond this limit, a positive (retards UTC) or a negative (advances UTC) leap second is added to it, usually at the end of June or December. By definition, UTC differs only from TAI by an integer number of seconds, they have the same rate. In 1972, TAI-UTC=10s, while now, TAI-UTC=32 seconds. This keeps the solar noon at the same UTC (averaged over the year). Leap seconds are given by the International Earth Rotation Service (IERS Bulletin C).

$$UT1 = UTC + DUT1 \, (\text{from IERS Bulletin D})$$

$$UTC = TAI - (\#\text{leap seconds})$$

GMST = Greenwich Mean Sideral Time. It is connected to sideral time, as it measures the Earth rotation with respect to distant celestial objects. A ***sideral year*** lasts approximately 366.25 days. It is the Greenwich hour angle of the mean equinox of date, measured in the true equator of date. It already takes into accounts Earth precession effects. Using the number of julian centuries (T=(JD-2451545.0)/36525) from 01/01/2000 12h00min00sec UT1:

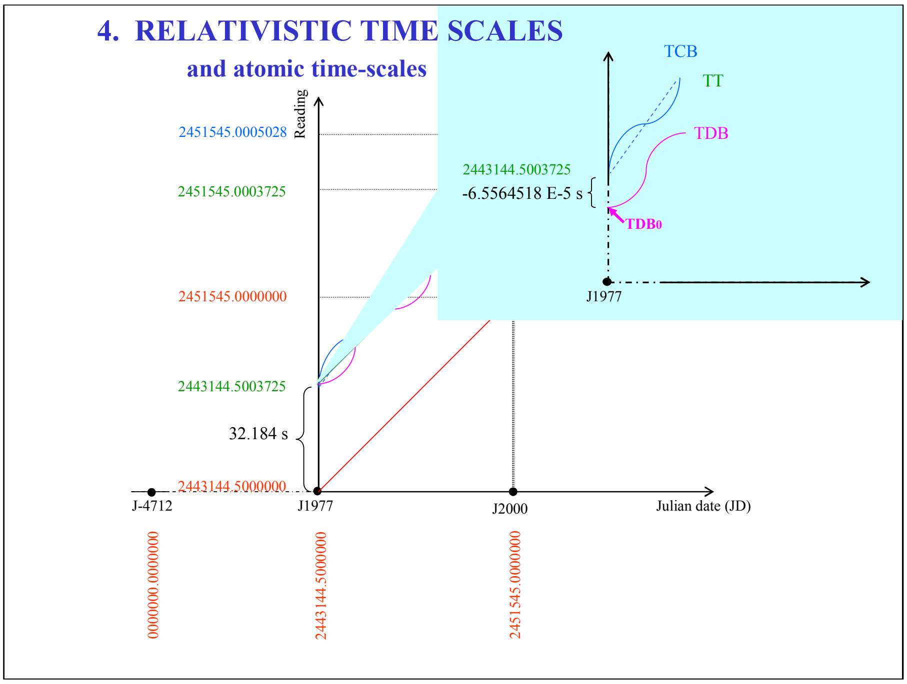
GAST = Greenwich Apparent Sideral Time. It is the GMST corrected for the shift in position of the vernal equinox due to nutation of the Earth pole.

$$GMST(\text{seconds at } UT1 = 0) = +24110.54841 + 8640184.812866 \cdot T + 0.093104 \cdot T^2 - 0.0000062 \cdot T^3$$

LMST = Local Mean Sideral Time. It is GMST plus the observer 's longitude measured positive to the east of Greenwich.

$$GAST = GMST + (\text{equation of the equinoxes})$$

$$LMST = GMST + long$$



1955: advent of atomic time.

TAI = International Atomic Time. It is based on the SI second (International System). The SI (International System) second is defined as the duration of 9 192 631 770 periods of radiation corresponding to the transition between two hyperfine levels of the ground state of the cesium-133 atom, considered on the geoid (mean sea level). TAI is an ideal time that represents what should measure an ideal atomic clock on the surface of the Earth ***rotating geoid*** (=sea level). TAI is obtained from worldwide system of synchronize atomic clock: it is calculated as a weighted average of times obtained from individual clocks with corrections applied for known relativistic effects (altitude above the geoid...) and environmental noise.

GPS or TPX = GPS or TOPEX master times. They use the SI second and differ from TAI only by a constant integer number of seconds (no leap second). It is TAI + average relativistic correction for the altitude of the satellite.

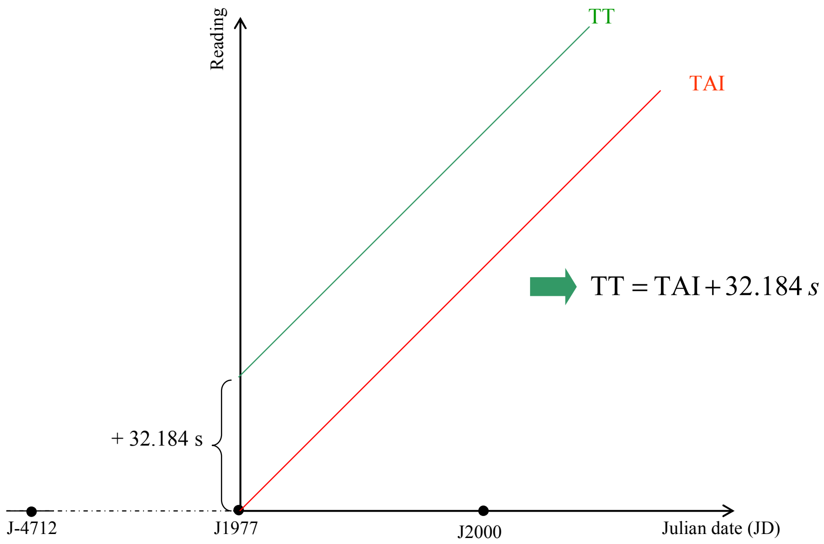
TT = Terrestrial Time (previously called TDT = Terrestrial Dynamical Time). It was created to be in continuity with the old ET (Ephemeris Time) used in planetary ephemerides before atomic clocks existed, and it replaced ET in 1984. It is an approximation of the proper time of clocks at rest on the ***non rotating geoid*** and it must be related to TCG by a constant difference in rate.

TCG = Geocentric *Coordinate* Time. It is the time coordinate associated with the relativistic Geocentric Coordinate Reference System (GCRS), centered upon the Earth, and non rotating. GCRS is known to be non inertial, because of the Earth ’s motion in the gravitational field of Solar System bodies; hence, its corresponding time scale is not uniform.

TCB = Barycentric *Coordinate* Time. It is the time coordinate associated with the relativistic Barycentric Coordinate Reference System (BCRS), centered upon the barycenter of the Solar System. BCRS is assumed to be inertial (cosmological effects are ignored); hence this time scale is uniform.

TDB = Barycentric *Dynamical* Time. It is the time presently used in the JPL (USA), EPM (Russia) or VSOP (France) planetary ephemerides. It is not based upon the SI second (there is a scale factor), and it is not a coordinate time. Eventually, TCB should replace TDB as the reference scale in the planetary ephemerides, as suggested by the IAU2000 recommendations.

a) TAI, TT:



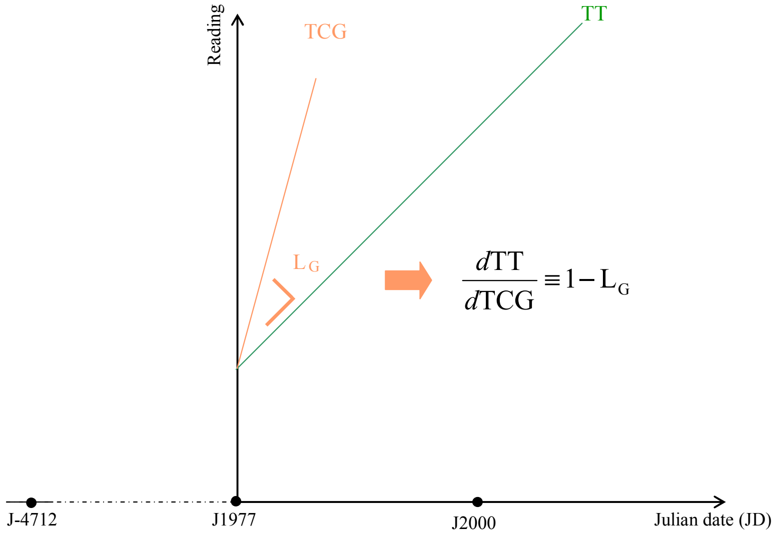
J2000= 01/01/2000 12h00'00''

J1977= 01/01/1977 00h00'00''

J-4712= 01/01/2000 12h00'00''

1
3

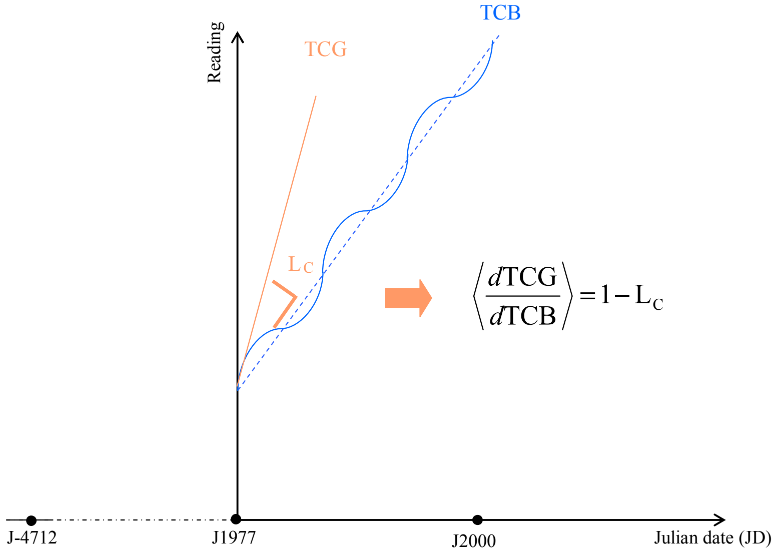
b) TCG, TT :



Reading
TCG
TT
L_G
$\frac{dTT}{dTCG} \equiv 1 - L_G$
J-4712
J1977
J2000
Julian date (JD)

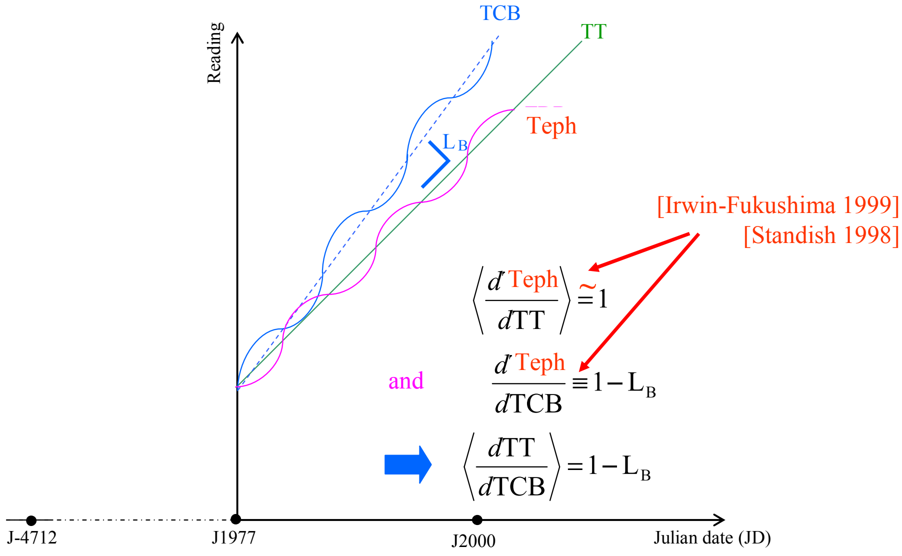
14

c) TCG, TCB :



$$\left\langle \frac{dTCG}{dTCB} \right\rangle = 1 - L_C$$

d) TCB, TT, TDB :


TCB
TT
Teph
L_B
Reading
[Irwin-Fukushima 1999]
[Standish 1998]
$\left\langle \frac{d\,\mathrm{Teph}}{d\mathrm{TT}} \right\rangle \tilde{=} 1$
and
$\frac{d\,\mathrm{Teph}}{d\mathrm{TCB}} \equiv 1 - L_B$
$\left\langle \frac{d\mathrm{TT}}{d\mathrm{TCB}} \right\rangle = 1 - L_B$
J-4712
J1977
J2000
Julian date (JD)

6. EXACT and APPROXIMATE TRANSFORMATIONS

a) in any reference frame :

Let $x^\mu = (ct, x, y, z)$ be coordinates, either BCRS or GCRS,

$\mathrm{K} \equiv \dfrac{1}{1-\mathrm{L_B}}$ be a scale factor (K≠1 for NON SI units) :

$$c^2\, d\tau^2 = ds^2 = \mathrm{K}^2\; g_{\mu\nu}(x^\mu, x^\nu)\; dx^\mu dx^\nu$$

$$\Longleftrightarrow \quad \int_{\tau_0}^{\tau} d\tau = \frac{\mathrm{K}}{c} \int_{t_0}^{t} \sqrt{g_{\mu\nu}(x^\mu, x^\nu)\, \frac{dx^\mu}{\mathrm{d}t} \frac{dx^\nu}{\mathrm{d}t}}\; \mathrm{d}t$$

at order $\Theta(\frac{1}{c^3})$

$$\tau - t \cong (\mathrm{K}-1)(t - t_0) - \frac{\mathrm{K}}{c^2} \int_{t_0}^{t} \left[W(x,y,z) + \frac{v^2}{2}(x,y,z) \right] \mathrm{d}t$$

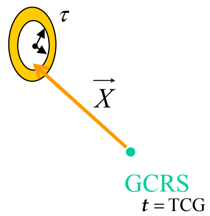
…and K=1 for t = TAI, TT, TCG, TCB

…but K≠1 for t = TDB

At order 1/c4, the integral is a function of the theory of gravitation used (all PPN parameters are equal to one in the case of GR).

We kept for W and Wi the GR expressions according to the IAU recommandations.

b) in GCRS, in terms of TCG scale :



$$W(\vec{X}) \equiv W_E(\vec{X}) + W_{ext\,E}(\vec{X})$$

- $W(\vec{X})$: Potential at clock
- $W_E(\vec{X})$: Potential at clock **due to Earth (E)**
- $W_{ext\,E}(\vec{X})$: Potential at clock **due to other bodies**

$$= W_{tidal\,E}(\vec{X}) + W_{iner\,E}(\vec{X})$$

- $W_{tidal\,E}(\vec{X})$: Earth tides **due to other solar system bodies (E)**
- $W_{iner\,E}(\vec{X})$: GCRS is non inertial

at order $\Theta(\frac{1}{c^3})$

$$\tau - \mathrm{TCG} \cong -\frac{1}{c^2}\int_{t_0=0}^{t=TCG}\left[W_{\mathrm{E}}(\vec{X}) + W_{tidal\,E}(\vec{X}) + \frac{V^2}{2}\right]\mathrm{d}t$$

K=1 , $\mathrm{TCG}_0 = 0$

At order 1/c², in the preceeding expression for dtau/dTCG, the contribution of terms in 1/c² reaches 7 E-10 for a clock on Earth or on a geostationary orbit.

In dtau/dTCG, the contribution from the inertial potential (Winer) times 1/c² is at max of about 2 E-20 for a clock on Earth or on a geostationary orbit. Hence it is neglected.

At order 1/c4, in the preceeding expression for dtau/dTCG, the contributions from the vector potential (Wi) and the square of the scalar potential (W²) amount at the maximum to 5 E-19 for a clock on Earth or on a geostationary orbit.

Hence, the given approximation at 1/c² for tau-TCG is sufficient for satellites within 50 000km from the center of the Earth (geostationary orbits) for present clock accuracies.

About the sideral trend value L_G **...**

Taking the previous expression at $\tau = TT$ (clock on geoid) gives

at order $\Theta\left(\frac{1}{c^3}\right)$

$$TT - TCG \cong -\frac{1}{c^2}\left[W_E\left(\vec{X}\right) + W_{tidal\ E}\left(\vec{X}\right)\right]\Big|_{geoid} TCG$$

U_{geoid}

$\left[W_E\left(\vec{x}\right) + W_{tidal\ E}\left(\vec{x}\right)\right]\Big|_{geoid} \approx$ cst for TCG

BUT: • U_{geoid} mesured with best precision of ~1m²/s² ➡ problem for precision E-17 in rate

• geoid difficult to define ➡ problem to calculate difference of potentiel at clocks above geoid

• geoid = fct of time

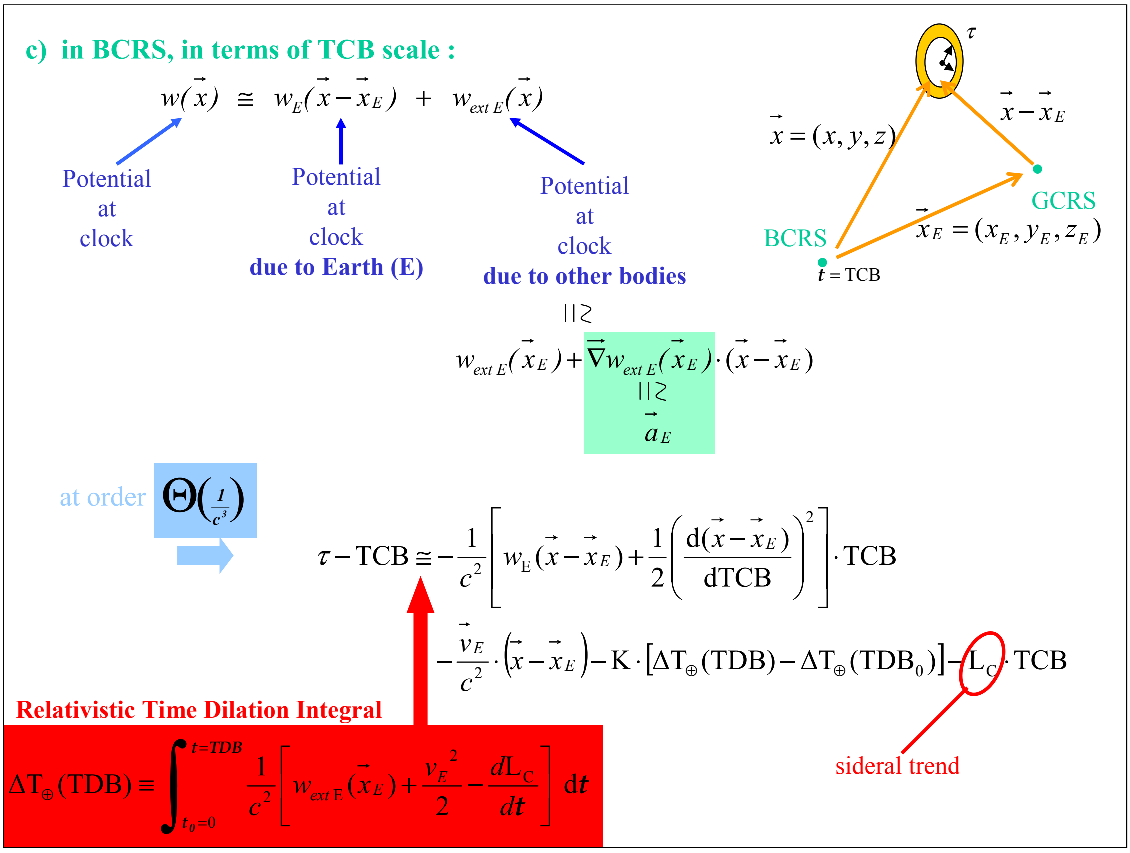
THUS: resolution B19 [IAU 2000]

1) $TT - TCG \equiv -L_G \cdot TCG$

sideral trend

L_G is a fixed value chosen to insure continuity with previous work.

The IAU resolution causes either the geoid to be a definition, not something physical that is measured, or it can be seen as defining TT as an artificial time, which would deviate from the proper time of clocks on the geoids at higher orders.



The order 1/c4 of the proper versus coordinate time transformation should not be neglected in the BCRS frame (unlike in the GCRS frame) according to present clocks accuracy.

Indeed, it reaches a maximum of 1 E-12 s near the geoid. For a range < 50 AU, it has a contribution to ranging and doppler effects of a maximum of about 1 E-6 m and 1 E-10 m/s respectively. But for simplicity, we will consider in the following transparencies only order $1/c^2$, and talk about the corresponding 1/c4 correction later on.

Remark: $\vec{X} \neq \vec{x} - \vec{x}_E$ because in relativity, there exists additional terms (for example: Lorentz transformation term)

Why is this sideral trend the L_C value?

Because taking the previous expression at $\tau = \text{TCG}$ gives

at order $\Theta\left(\frac{1}{c^3}\right)$

$$\text{TCG} - \text{TCB} \cong -\frac{\vec{v}_E}{c^2} \cdot \left(\vec{x} - \vec{x}_E\right) - \text{K} \cdot \left[\Delta\text{T}_{\oplus}(\text{TDB}) - \Delta\text{T}_{\oplus}(\text{TDB}_0)\right] - \text{L}_{?} \cdot \text{TCB}$$

Topocentric terms:
only periodic

Relativistic Time Dilation Integral:
only periodic
because sideral trend $\text{L}_{?}$
removed by definition

and $$\left\langle \frac{d\text{TCG}}{d\text{TCB}} \right\rangle = 1 - \text{L}_C$$

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the value of $\mathrm{K} \equiv \dfrac{1}{1-\mathrm{L_B}} = \dfrac{1}{(1-\mathrm{L_G})(1-\mathrm{L_C})}$

$\dfrac{d\mathrm{TT}}{d\mathrm{TCG}} \equiv 1-\mathrm{L_G}$ and same common epoch for TT, TCG

⇔ $\mathrm{TDB}-\mathrm{TT} = \mathrm{TDB} - (1-\mathrm{L_G})\cdot \mathrm{TCG}$

⇒ $\mathrm{TDB}-\mathrm{TT} \cong \mathrm{TDB} - (1-\mathrm{L_G})\left[(1-\mathrm{L_C})\cdot \mathrm{TCB} - \dfrac{\vec{v}_E}{c^2}\cdot\left(\vec{x}-\vec{x}_E\right) - \mathrm{K}\cdot\left[\Delta\mathrm{T}_\oplus(\mathrm{TDB}) - \Delta\mathrm{T}_\oplus(\mathrm{TDB}_0)\right]\right]$

⇒ $\mathrm{TDB}-\mathrm{TT} \cong \mathrm{TDB}_0 + (1-\mathrm{L_G})\cdot\dfrac{\vec{v}_E}{c^2}\cdot\left(\vec{x}-\vec{x}_E\right)$

$+ (1-\mathrm{L_G})\cdot \mathrm{K}\cdot\left[\Delta\mathrm{T}_\oplus(\mathrm{TDB}) - \Delta\mathrm{T}_\oplus(\mathrm{TDB}_0)\right]$

d) in BCRS, in terms of TDB scale :

Consequences of « K »:

	… on TT, TCB, TCG scales … on TDB scale
Time	K
Position	K
GM	K
GM/distance	1
v	1

K = scale factor, is called « L » in Moyer ’s monograph [Moyer 2000].

masses, positions and velocities taken from planetary ephemeris

2) $$\tau - \mathrm{TCB} \cong -\mathrm{L_C} \cdot \mathrm{TCB} - \frac{\boxed{\mathrm{K}}}{c^2}\left[w_{\mathrm{E}}(\vec{x} - \vec{x}_E) + \frac{1}{2}\left(\frac{\mathrm{d}(\vec{x} - \vec{x}_E)}{\mathrm{dTCB}}\right)^2 \right] \cdot \mathrm{TCB}$$

$$- \boxed{\mathrm{K}} \frac{\vec{v}_E}{c^2} \cdot (\vec{x} - \vec{x}_E) - \mathrm{K} \cdot [\Delta \mathrm{T}_{\oplus}(\mathrm{TDB}) - \Delta \mathrm{T}_{\oplus}(\mathrm{TDB}_0)]$$

3) $$\mathrm{TCG} - \mathrm{TCB} \cong -\mathrm{L_C} \cdot \mathrm{TCB} - \boxed{\mathrm{K}} \frac{\vec{v}_E}{c^2} \cdot (\vec{x} - \vec{x}_E) - \mathrm{K} \cdot [\Delta \mathrm{T}_{\oplus}(\mathrm{TDB}) - \Delta \mathrm{T}_{\oplus}(\mathrm{TDB}_0)]$$

4) $$\mathrm{TDB} - \mathrm{TT} \cong \mathrm{TDB}_0 + (1 - \mathrm{L_G}) \cdot \boxed{\mathrm{K}} \cdot \frac{\vec{v}_E}{c^2} \cdot (\vec{x} - \vec{x}_E)$$

$$+ (1 - \mathrm{L_G}) \cdot \mathrm{K} \cdot [\Delta \mathrm{T}_{\oplus}(\mathrm{TDB}) - \Delta \mathrm{T}_{\oplus}(\mathrm{TDB}_0)]$$

5) $$\mathrm{TDB} - \mathrm{TDB}_0 = (1 - \mathrm{L_B}) \cdot \mathrm{TCB}$$

! **can safely neglect** $\Delta \mathrm{T}_{\oplus}(\mathrm{TDB}_0)$

and assume $\Delta \mathrm{T}_{\oplus}(\mathrm{TDB}) \cong \Delta \mathrm{T}_{\oplus}(\mathrm{TT})$

In the expression Tau-TCB, for a clock on Earth, the maximum consequences of the third term are

a) 74.3 m/UA + 51m on range measurements

b) 1.72 E-3 m/s/AU + 42 E-3 m/s on doppler two-ways measurements

c) …those effects are bigger than those for the three-ways measurements for a range<50 AU; but three-ways corrections can be considerably more important for ranges above 50 AU.

In the expression TCG-TCB, normaly, at the requiered precision (0.2 E-12 s in amplitude), we should consider terms up to 1/c4 included. This means some clock position dependent terms, which reach a maximum of order 0.4 E-12 s on a geostationary orbit.

$$-\frac{\vec{v}_E}{c^4} \cdot (\vec{x} - \vec{x}_E)\left[3\,\mathrm{W}_{0\,\mathrm{ext\,E}}(\vec{x}_E) + \frac{\vec{v}_E^{\,2}}{2}\right]$$

Expressions 1), 2), 3), 4) and 5) alltogether provide the transformations between the different relativistic time scales with the required precision.

e) Sideral trends :

«L_C is determined by finding the zero of the slope of the residuals between the numerical time ephemerides which approximate the integral and corrected secular + sinusoidal series (with origin at J2000 but no linear term) for the time ephemeris. (…) The exact value of L_C depends on the underlying planetary and lunar ephemeris used to create a time ephemeris »

[Irwin-Fukushima 1999]

« Because no unambiguous definition may be provided for L_B and L_C, these constants should not be used in formulating transformations when it would require knowing their value with an uncertainty of order E-16 or less »

[IAU 2000 .B1.5 note 3]

	REFERENCES	VALUES	COMMENTS
L_C	[Irwin-Fukushima 1999]	1.48082686741 E-8 +/- 2 E-17	**Calculated from planetary ephemeris :** (IAU2000 resolutions)
L_G	[Soffel et al. 2003]	6.969290134 E-10	**Fixed value :** Definition (IAU2000 resolutions)
L_B	[Soffel et al. 2003]	1.5505051976772 E-8 +/- 3 10 E_17	**Deduced from L_C and L_G :** L_B=L_C+L_G - L_C . L_G (IAU2000 resolutions)

∋ average of order $\Theta(\frac{1}{c^5})$

∋ Asteroids perturbations

[Irwin-Fukushima 1999] take a different value for

L_G=6.969290112 E-10 +/- 6 E-18 [Bursa et al.1997],

calculated as the value of the potential on the geoid (sea level) divided by c^2.

Hence they obtain a different value for

L_B=1.55051976749 E-8 +/- 3 E-17.

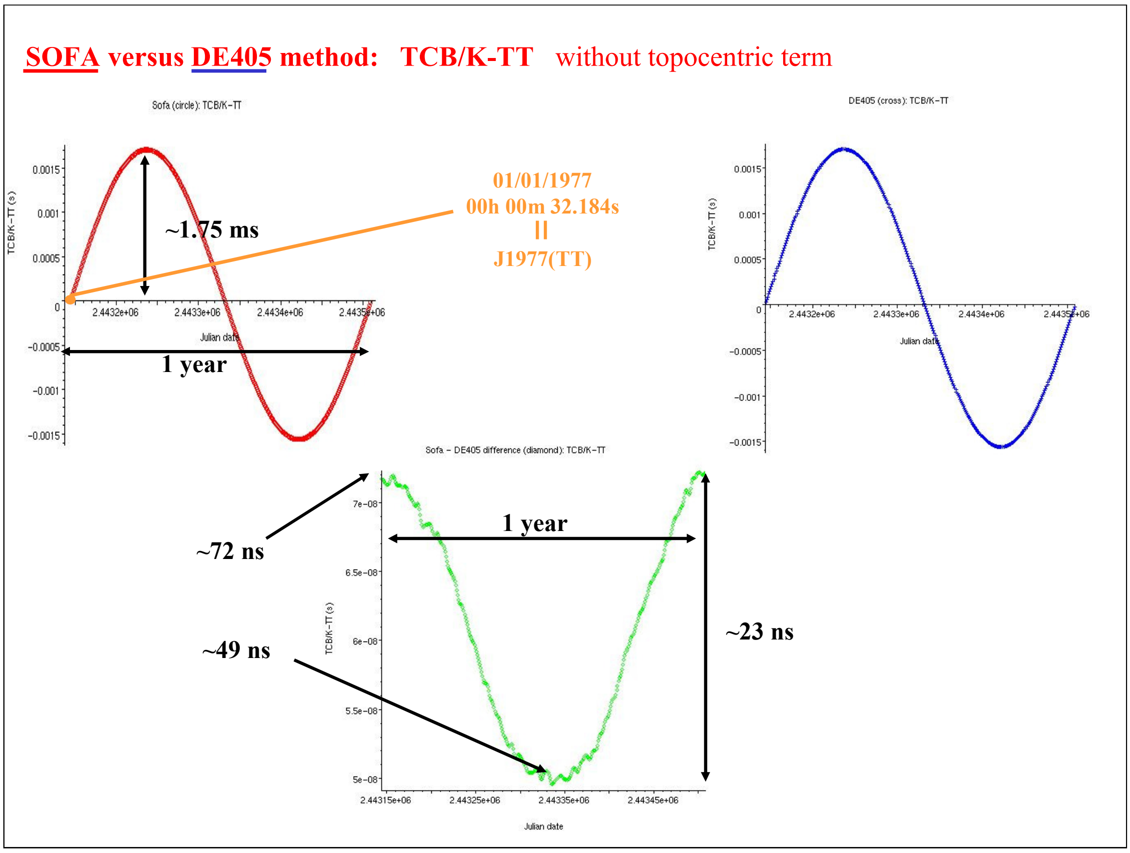
But their article was published before the IAU2000 resolution that fixed L_G as a definition.

f) Computing the Relativistic Time Dilation Integral :

Different methods:

REFERENCES	METHOD	PLANETARY EPHEMERIS	PRECISION (QUOTED !!)	VALIDITY	COMMENTS
[Thomas 1975] [Moyer 1981] [Moyer 200]	**Analytical** developpement of integral (keplerian orbits)		- 20 µs for 6 coef.		- Periodic variations of Moon-Earth = large errors - Vector formula more accurate
[Hirayama et al. 1988]	**Analytical** developpement of integral		- 5 ns for 131 coef		
[IAU 2003] [Fairhead-Bretagnon 1990]	Fairhead-Bretagnon **analytical** model (series)	- VSOP82 - ELP 2000 …ajusted to DE200	- 100 ns for 127 coef. - 1ns for 750 coef. - < 3 ns for 787 coef. (SOFA)	1000 years around J2000	FB **series** corrected for - masses to agree with JPL - 3 coefficients were wrongly divided by (1-L_C)
[Irwin-Fukushima 1999]	**Numerical** integration and tchebytchev tabulation	- DE200 Or - DE405	- ~ 0.1ns	1600-2000	**Time ephemeris** : - TE200 Or - TE405

Precision for Moyer ???



Zero point error???

Conclusions

- Relativity must be taken into account

- Reference frames need to be carefully defined:

  space (rotating ?), time, transfo BCRS-GCRS-MoRS

  => IAU 2000 conventions

-Time tags not to be confused with time coordinates

-Need carefull description of time transfo at appropriate accuracy

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