

Light deflection in Weyl gravity: constraints on the linear parameter

Sophie Pireaux¹

UMR 5562, Dynamique Terrestre et Planétaire (DTP), B105 Observatoire Midi-Pyrénées,
14 Avenue Edouard Belin, 31400 Toulouse, France

E-mail: sophie.pireaux@cnes.fr

Received 5 February 2004

Published 24 August 2004

Online at stacks.iop.org/CQG/21/4317

doi:10.1088/0264-9381/21/18/004

Abstract

Light deflection offers an unbiased test of Weyl's gravity since no assumption on the conformal factor needs to be made. In this second paper of our series '*light deflection in Weyl gravity*', we analyse the constraints imposed by light deflection experiments on the linear parameter of Weyl's theory. Regarding solar system experiments, the recent CASSINI Doppler measurements are used to infer an upper bound, $\sim 10^{-19} \text{ m}^{-1}$, on the absolute value of the above Weyl parameter. In non-solar system experiments, a condition for unbound orbits together with gravitational mirage observations enables us to further constrain the allowed negative range of the Weyl parameter to $\sim -10^{-31} \text{ m}^{-1}$. We show that the characteristics of the light curve in microlensing or gravitational mirages, deduced from the lens equation, cannot be recast into the general relativistic predictions by a simple rescaling of the deflector mass or of the ring radius. However, the corrective factor, which depends on the Weyl parameter value and on the lensing configuration, is small, even perhaps negligible, owing to the upper bound inferred on the absolute value of a negative Weyl parameter. A statistical study on observed lensing systems is required to settle the question. Our Weyl parameter range is more reliable than the single value derived by Mannheim and Kazanas from fits to galactic rotation curves, $\sim +10^{-26} \text{ m}^{-1}$. Indeed, the latter, although consistent with our bounds, is biased by the choice of a specific conformal factor.

PACS numbers: 04.25.Nx, 04.50.+h, 04.80.Cc, 04.90.+e, 95.30.Sf, 95.35.+d

(Some figures in this article are in colour only in the electronic version)

¹ Previously working at: Unité de Physique Théorique et Mathématique (FYMA), Université catholique de Louvain (UCL), Belgium.

1. Introduction

As explained in [Pi2004] an alternative theory of gravitation is highly desirable. Indeed, solely from a theoretical point of view, the choice of the Hilbert–Einstein action is not based on any fundamental principle, Einstein’s theory of gravitation cannot be properly described by quantum field perturbation theory, neither is it invariant under conformal transformations. Those latter two lacks make it difficult to unify gravitation with other fundamental interactions. From the point of view of experiments, the Newtonian potential recovered in the weak field regime of general relativity cannot reproduce the flat velocity distributions in the vicinity of galaxies without copious amounts of dark matter.

Regarding the demand for an alternative theory, and among many candidates, the Weyl theory is an interesting prototype. Not only is it conformally invariant, but it contains an additional linear contribution to the Newtonian potential. This latter feature is encoded in the key parameter of the theory, γ_W . The Weyl gravity holds a total of three free parameters: γ_W , β_W and k_W ; plus a conformal factor which is to be specified by a consistent study of the coupling of the Weyl gravity to matter fields. To recover a Newtonian potential for photons on short distance scales, the second parameter is constrained to $\beta_W(M) = \frac{G_N M}{c^2}$, where G_N is the Newtonian constant, M the total gravitational mass (luminous or not) and c is the speed of light. General relativity then corresponds to the particular case $\gamma_W = k_W = 0$. The parameter k_W should be effective only on cosmological distance scales. Nevertheless, as shown in [Pi2004], photon paths are insensitive to k_W , as well as to the unknown conformal factor. Hence, light deflection experiments offer an interesting tool to constrain the Weyl linear parameter γ_W . In the following approach, we shall call it the Weyl parameter and neglect the mixed ($\beta_W \gamma_W$)-term when compared with those of γ_W and β_W alone.

In [Pi2004], we inferred criteria for light deflection to take place and introduced critical distances in Weyl gravity regarding photon trajectories. Let us recall the relevant ones, namely: the distance that separates unbound and bound orbits,

$$r_{\text{null}} \sim -\frac{1}{\gamma_W}, \quad (1)$$

which is physical for negative values of the Weyl parameter; and the critical closest approach distance separating the convergent and the divergent regime,

$$r_{00} \sim \sqrt{6} \times 10^{\frac{x+3}{2}} \frac{1}{\sqrt{|\gamma_W|}} \text{ m} \quad \text{for } M = 10^x M_{\text{Sun}}, \gamma_W \text{ in } (\text{m}^{-1}), \quad (2)$$

when the Weyl parameter is positive. As in [Pi2004], our present analysis will be carried out consistently with respect to the weak field

$$r_{\text{weak field}} \ll \frac{1}{|\gamma_W|} \quad (3)$$

and the strong field

$$r_{\text{strong field}} \gg \sqrt{3} \times 10^{\frac{x+3}{2}} \frac{1}{\sqrt{|\gamma_W|}} \text{ m} \quad \text{for } M = 10^x M_{\text{Sun}}, \gamma_W \text{ in } (\text{m}^{-1}) \quad (4)$$

approximations previously introduced. Indeed, together with the critical radii for photons, those limiting distances are functions of the linear parameter γ_W .

Keeping those key distances in mind, we shall now confront predictions of Weyl theory regarding light deflection with observations in order to constrain γ_W . Historically, the spectacular phenomenon of light deflection by a gravitational source (namely the Sun) was the trigger to the success of Einstein’s theory. Today, the arrival of new detection techniques allows not only for precise measurement of the light deflection angle (change in the apparent

position of a light source) due to the Sun or planets in our solar system, but also for observation of microlensing events (variation of the received light flux) at the scale of our galaxy, and of gravitational mirages (multiple images) or weak lensing (distorted images) at extragalactic distance scales. So light deflection experiments allow us to test the universality of relativistic theories of gravitation over different distance scales by considering successively a close star, a galaxy or a distant quasar as light sources lensed by a solar system body, a star of the galactic halo, a galaxy or a cluster. Doing so, they also explore very different mass scales for the lens.

2. Solar system experiments

The present section of this paper aims at a confrontation of Weyl gravity with solar system experiments. In the previous article ‘*Light deflection in Weyl gravity: critical distances for photon paths*’ by Pireaux [Pi2004], we obtained the expression for the asymptotic light deflection angle in the weak field regime (equation (18) in [Pi2004]). Neglecting the $(\beta_W \gamma_W)$ -contribution in the Weyl gravitational potential (equation (4) in [Pi2004]), we find

$$\hat{\alpha}_{\text{weak field}}(r_0) \simeq +\frac{4\beta_W}{r_0} - \gamma_W r_0, \quad (5)$$

where r_0 is the closest approach distance of the photon to the deflector. The prediction about the weak field deflection angle already differs from the Einsteinian one as soon as γ_W is nonzero.

The light deflection angle due to the gravitational field of the Sun was the very first prediction of general relativity which originally confirmed this theory within a 20% error margin. Today, with modern techniques operating in the radio-waveband, the precision has reached about 0.001% [Be *et al* 2003], allowing the planets of our solar system to be considered as potential deflectors too. At first order, deviations from general relativity regarding light deflection are encoded in the post-Newtonian (PN) parameter γ (not to be confused with the Weyl parameter γ_W), with $\gamma = 1$ for general relativity.

2.1. Estimation of γ_W from VLBI data

We use first-order light deflection measurements provided by very large baseline interferometry (VLBI), more precisely measurements of the corresponding time delay due to photons being deflected by the Sun, to obtain constraints on the Weyl parameter γ_W .

The measurements dedicated to γ made in 1995 using quasars 3C273 and 3C279 [Le *et al* 1995] constrained the post-Newtonian parameter γ to

$$\gamma = 0.9996 \pm 0.0017. \quad (6)$$

Now, VLBI type II experiments are in progress, no longer dedicated to light deflection, but providing an indirect measurement of γ , while monitoring polar motion and Earth rotation. The project has grown into a network of more than 87 observatories and is sensitive to light deflection over almost the entire celestial sphere. An analysis of over 1.7 million ionosphere-corrected group delay measurements involving 541 radio sources [Sh *et al* 2004] leads to the value of²

$$\gamma = 0.99983 \pm 0.00045. \quad (7)$$

² An analysis of over 2 million observations quoted by Will [Wi2001] from [Eu *et al* 1999] leads to $(1 + \gamma)/2 = 0.99992 \pm 0.00014$. But [Eu *et al* 1999] is unpublished. Other less stringent estimates of γ were obtained in the past from VLBI data, see [Se *et al* 1970, Co *et al* 1974, FoSr1976, RoCa1984, Ro *et al* 1991a, Ro *et al* 1991b].

We now assume β_W given by $\frac{G_N M_{\text{Sun}}}{c^2}$ to recover the Newtonian potential in the solar system weak field limit, and extrapolate the VLBI results for light deflection at the solar limb. Matching the first-order post-Newtonian expression for the light deflection angle with that of the Weyl theory (5) leads to³

$$\gamma_W = \frac{(1-\gamma)}{2} \frac{4G_N M_{\text{Sun}}}{R_{\text{Sun}} c^2} \frac{1}{R_{\text{Sun}}} \quad (8)$$

$$\Downarrow$$

$$\begin{cases} -7.9 \times 10^{-18} \text{ m}^{-1} \leq \gamma_W \leq 1.3 \times 10^{-17} \text{ m}^{-1} & \text{for (6),} \\ -1.7 \times 10^{-18} \text{ m}^{-1} \leq \gamma_W \leq 3.8 \times 10^{-18} \text{ m}^{-1} & \text{for (7).} \end{cases}$$

This estimation with recent VLBI data provides a range of values for γ_W which contains the particular order of magnitude needed by Mannheim and his collaborators to fit the galactic rotation curves [Ma1994, Ma1995, MaKa1989], namely

$$\gamma_{W\text{Mannheim-Kazanas}} \sim +10^{-26} \text{ m}^{-1}. \quad (9)$$

The range (8) is narrower than that given by Edery and Paranjape [EdPa1998], based on an estimation of the PN parameter given in an article of 1976.

2.2. Estimation of γ_W from Cassini data

The Cassini experiment was carried out between 6 June and 7 July 2002. The spacecraft was on its way to Saturn. Measurements were made around the time of solar conjunction, when the spacecraft was almost aligned with the Sun and the Earth, that occurred on 21 June 2002. Those new constraints on the PN parameter γ were obtained with a Doppler method. Motion of the spacecraft produces a change in the time delay of light transmitted between the spacecraft and the Earth, as well as in its impact parameter respective to the Sun. These are equivalent to a change in the distance and hence in the relative radial velocity between the spacecraft and the Earth, resulting in a Doppler effect. The Doppler method coupled to a new radio configuration using double-band and multifrequency link allowed the constraints on γ to be increased by one order of magnitude [Be *et al* 2003]. The constraints we infer on the Weyl parameter are correspondingly improved:

$$\gamma - 1 = (-2.1 \pm 2.3) \times 10^{-5} \quad (10)$$

$$\Downarrow \quad (11)$$

$$-1.2 \times 10^{-20} \text{ m}^{-1} \leq \gamma_W \leq 2.7 \times 10^{-19} \text{ m}^{-1} \quad \text{for (10).}$$

2.3. Sign of γ_W ?

There exist planned experiments which should improve measurements of the post-Newtonian parameter γ with a precision of $\sim 5 \times 10^{-7}$, such as the future GAIA mission intended to measure γ as a by-product of microarcsecond-astrometry [GAIA2000]; or the project LATOR, dedicated to γ , which should reach a precision of $\sim 5 \times 10^{-8}$ [Tury *et al* 2004]. Although higher precision tests on light deflection could reduce the allowed range of values for γ_W , the test of light deflection in the neighbourhood of the Sun cannot help us to decide on the sign of the parameter γ_W .

Considering that the light deflection angle calculated with general relativity for the visible gravitational mass in galaxies or clusters at galactic distance scales is often inferior to the

³ Even though, strictly speaking, the Weyl gravity does not allow for a post(-post) Newtonian development, because the corresponding potential for photons diverges on asymptotical radial distances.

observed deflection, the presence of gravitational dark matter is inferred. So, if one wishes the linear γ_W -term of the Weyl potential to be an alternative to (a too large amount of) dark matter contributing to light deflection, then $-\gamma_W r_0$ must be positive. However, this sign of γ_W is just the opposite of the sign argued by Mannheim and Kazanas in their parametrization (9). Two types of arguments might be given in order to solve this apparent contradiction and prevent ruling out the Weyl theory.

A first possibility is to consider γ_W to be positive, so that the theory would still need the ‘magic’ contribution of dark matter to explain light deflection due to galaxies and clusters, just like general relativity does. This possibility leads to divergent deflection (a negative $\hat{\alpha}$ in equation (5)) on radial closest approach distances from the deflector larger than $r_{00}(M, \gamma_W)$ (2), where the linear divergent contribution becomes dominant. The positive sign was considered in our graduate thesis work [Pi1997], without dark matter.

An alternative argument would be to claim that tests of the Weyl theory taking into account massive particles or bodies are ambiguous, in comparison with those based on nonmassive particles such as photons, or ultra relativistic particles. Indeed, the presence of matter breaks the conformal symmetry of the theory, and this symmetry breaking mechanism is not well understood. In order to fit galactic rotation curves, we need to fix the arbitrary conformal factor of the line element (equation (3) in [Pi2004]) because massive geodesics are not conformally invariant. Mannheim and Kazanas (arbitrarily) chose the conformal factor $\chi^2(r) \equiv 1$ (or constant) and obtained $\gamma_W > 0$ through those fits. Their work corresponds to a particular theory:

$$\text{Mannheim–Kazanas theory} \equiv \left\{ \begin{array}{l} \text{Weyl theory} \\ + \\ \chi^2(r) \equiv 1 \\ + \\ \text{fits to galactic rotation curves (9)} \end{array} \right\} \quad (12)$$

However, the physical conformal factor in the spherically symmetric metric (equation (3) in [Pi2004]), specified by the symmetry breaking mechanism, could be different from a constant. Hence, the physical parameter γ_W present in the metric would be different from the estimate of Mannheim and Kazanas. For example, Edery *et al* [Ed *et al* 2001] have shown that, in the weak field limit (which applies to the galactic rotation-curve parametrization and to light deflection in the solar system), it is possible to find an appropriate conformal factor $\chi^2(r)$ and a radial coordinate transformation $r'(r)$, so as to change the sign of the γ_W -term in the Weyl gravitational potential $V_W(r, \gamma_W)$ given in [Pi2004], when the $(\beta_W \gamma_W)$ -term is neglected. One easily checks that, if the conformal factor is given by $\chi^2(r) \simeq 1 - 2\gamma_W r$ with $r' = \chi(r) \cdot r$, then

$$\begin{aligned} ds^2(r, \gamma_W) &\equiv \chi^2(r) \cdot \left\{ \begin{array}{l} + \left[1 + \frac{2V_W(r, \gamma_W)}{c^2} \right] c^2 dt^2 \\ - \left[1 + \frac{2V_W(r, \gamma_W)}{c^2} \right]^{-1} dr'^2 \\ - r^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \end{array} \right\} \\ &\simeq \left\{ \begin{array}{l} + \left[1 + \frac{2V_W(r', \gamma'_W)}{c^2} \right] c^2 dt^2 \\ - \left[1 + \frac{2V_W(r', \gamma'_W)}{c^2} \right]^{-1} dr'^2 \\ - r'^2 (d\theta^2 + \sin^2 \theta d\varphi^2) \end{array} \right\} \equiv ds^2(r', \gamma'_W) \end{aligned} \quad (13)$$

where $\gamma'_W \equiv -\gamma_W$.

In conclusion, until a conformal factor is specified by a consistent study of the coupling of Weyl gravity to matter fields, the conservative bounds deduced from solar system light deflection experiments (8), (11) are preferable. And if a negative sign is taken for γ_W for photons, then light deflection is always convergent.

3. Beyond solar system experiments

3.1. Constraints on a negative γ_W from the existence of gravitational mirages

We have shown in the article ‘*Light deflection in Weyl gravity: critical distances for photon paths*’, in particular through discussion (22) and figure 9 of [Pi2004], that there exists no asymptotic state for photons if the closest approach distance r_0 is larger than r_{null} (1), when the Weyl linear parameter γ_W is negative. Hence no asymptotic light deflection can take place. The existence of gravitational mirages with an Einstein angle ϑ_E (corresponding to the Einstein radius r_E) of a few arcseconds puts a stronger upper bound on the absolute value of the Weyl linear parameter. Indeed, using a rough estimation of the distance as a function of the redshift, based on the empirical Hubble law,

$$D \simeq \frac{c}{H_0} z \quad \text{for } z < 1, \quad (14)$$

we find

$$\frac{r_0}{D_{\text{ol}}} \sim \frac{r_E}{D_{\text{ol}}} \simeq \vartheta_E \lesssim \frac{r_{\text{null}}}{D_{\text{ol}}} \stackrel{(1)}{\Leftrightarrow} |\gamma_W| \lesssim \left[\frac{1}{\vartheta_E} h_0 \frac{0.3}{z_L} \right] 1.7 \times 10^{-31} \text{ m}^{-1} \quad \text{for } \gamma_W < 0$$

with $\left\{ \begin{array}{l} D_{\text{ol}}, \text{ the distance observer–lens,} \\ \vartheta_E, \text{ the Einstein angle corresponding to a mirage ring, in arcseconds,} \\ h_0, \text{ the normalized Hubble parameter } \in]0, 55; 0, 75[\\ \quad \text{with } H_0 = 100 h_0 \text{ km s}^{-1} \text{ Mpc}^{-1} \text{ [Turner 1999],} \\ z_L, \text{ the redshift of the lens.} \end{array} \right.$

(15)

Consequently, it seems reasonable to admit the following conservative limit on γ_W :

$$|\gamma_W| < 10^{-31} \text{ m}^{-1} \quad \text{for } \gamma_W < 0. \quad (16)$$

When γ_W is positive, in contrast, we cannot find a better upper bound than the conservative value obtained from solar system experiments (11). Otherwise, we have to adopt the Mannheim–Kazanas parametrization (9) based on an arbitrary constant value of the conformal factor.

3.2. Relevance of the weak field versus the strong field limit

If we wish to infer further constraints on the Weyl parameter from microlensing or gravitational lensing events, it is crucial to stick to the constraints so far available, respectively (16) for a negative parameter, or (11) for a positive parameter, and to respect the limits of the approximations introduced in [Pi2004] which are functions of γ_W . In fact, the weak (or strong) field limit on the radial distance measured from the gravitational lens has to be verified on the photon path, all the way from the light source to the observer. That is, the limit does not only apply to the lens–observer (D_{ol}) and lens–source (D_{ls}) distances, but also to the closest approach distance of the photon onto the lens (r_0). This distance is considered to be of the same order of magnitude as the Einstein ring radius (r_E) associated with the observer–lens–source (O–L–S) system.

Immediately, we see that *the strong field limit* (4) is of no use here, because we only have in hand an upper bound on $|\gamma_W|$. The lower bound is given by general relativity ($\gamma_W = 0$) and leads to a strong field limit only valid at infinity.

As far as the *weak field limit* (3) is concerned, we are limited by our upper bound on $|\gamma_W|$.

Our numerical example for a microlens is realized for a point-like lens (L) of one solar mass placed in the galactic cloud, and a stellar source (S) present in the Large Magellanic Cloud: when looking towards the halo of our Galaxy, an O–L–S microlensing system characterized by

$$D_{\text{os}} = 2 \times 10^{21} \text{ m}, \quad D_{\text{ol}} = 5 \times 10^{20} \text{ m}, \quad (17)$$

which imply

$$r_E = 1.5 \times 10^{12} \text{ m}, \quad (18)$$

and a typical angular separation between images of the order of a milliarcsecond. Alternatively, if one was considering microlensing towards the Galactic Bulge, the characteristic distances would be instead $D_{\text{os}} \sim 8 \text{ kpc} = 2.5 \times 10^{20} \text{ m}$ and $D_{\text{ol}} = D_{\text{os}}/2$. For a lens of one solar mass, this means an Einstein radius of $6.1 \times 10^{11} \text{ m}$. Microlensing by a point mass model is a good approximation of reality in the simple limiting case of small lensing probability along the line of sight. A microlensing event in the massive halo of the Milky Way is a good example of this. In contrast, if the line of sight passes through the centre of the galaxy, the lensing optical depth (lensing probability) may approach 1, requiring a more complex model with intricate mass contributions. In the case of microlenses in the dark halo, the lens speed is negligible with respect to the speed of light. Therefore, we ignore the frequency shift of the light due to the changing of path length along the lines of sight for the different images. Hence, the surface brightness is the same for all images, and the flux density is proportional to the solid angle of the images. This point allows easy computation of the image amplification. Owing to distance scales involved in microlensing events, the conservative estimate (11) from solar system experiments will not allow us to improve constraints on a positive parameter, because, strictly speaking, we are not allowed to use this weak field limit. In contrast, the more stringent bound (16) obtained for a negative parameter makes it possible to discuss microlensing predictions when γ_W is negative.

In the case of gravitational mirages, we shall use a point-like lens model representing either a galaxy ($M \sim 10^{11} M_{\text{Sun}}$) or a cluster of galaxies ($M \sim 10^{13} - 10^{14} - 10^{15} M_{\text{Sun}}$), with the following distance scales

$$D_{\text{ol}} = 10^8 \frac{G_N M}{c^2}, \quad D_{\text{os}} = 2 D_{\text{ol}}, \quad (19)$$

implying

$$r_E = \sqrt{2} \times 10^4 \frac{G_N M}{c^2} \simeq \begin{cases} 2 \times 10^{18} \text{ m for } M = 10^{11} M_{\text{Sun}} \\ 2 \times 10^{20} \text{ m for } M = 10^{13} M_{\text{Sun}} \\ 2 \times 10^{21} \text{ m for } M = 10^{14} M_{\text{Sun}} \\ 2 \times 10^{22} \text{ m for } M = 10^{15} M_{\text{Sun}} \end{cases} \text{ a cluster.} \quad (20)$$

Distance scales involved in gravitational mirages prevent us from using the weak field approximation with our bounds (11) on a positive γ_W , but a negative value of the parameter can be considered.

Whichever type of lensing event (microlensing or gravitational mirages) we consider, conditions for light deflection to take place ($r_0 < r_{\text{null}}$, [Pi2004]) are fulfilled each time we work in the weak field limit, according to the coincidence of $r_{\text{weak field}}$ (3) and r_{null} (1).

3.3. Constraints on a negative γ_W

With the above remarks in mind, we now extract information from microlensing events or gravitational mirages for a negative value of the linear Weyl parameter.

3.3.1. Asymptotic weak field light deflection angle. In the weak field limit, using $r_0 \simeq D_{\text{ol}} \sin \vartheta_I$ at zero order in $V_W(r)/c^2$, equation (5) can be rewritten, at first order, as

$$\hat{\alpha}_{\text{weak field}}(\vartheta_I, D_{\text{ol}}) \simeq + \frac{4\beta_W}{D_{\text{ol}} \sin \vartheta_I} - \gamma_W D_{\text{ol}} \sin \vartheta_I \quad (21)$$

where ϑ_I is the angular position of the observed image (I), with respect to the O–L axis (see figure 1). The Weyl deflection angle in the weak field limit cannot be rescaled to the Einsteinian prediction by a redefinition of the mass, as is the case in tensor scalar theories. So, predictions of Weyl gravity are expected to be qualitatively different from the general relativistic ones.

3.3.2. Lens equation in the weak field limit. The lens equation associated with a converging deflection angle (21) in the weak field limit and small angle approximation ($\vartheta_I, \vartheta_S, \hat{\alpha} \ll \sqrt{3}$ rad) is

$$\vec{\vartheta}_I^2 - \frac{\vec{\vartheta}_S}{1+n_W} \vec{\vartheta}_I - \vartheta_W^2 = 0, \quad (22)$$

where we have defined

$$\begin{aligned} \vartheta_E &\equiv \text{angular radius of the Einstein ring} \\ &\equiv \sqrt{\frac{4G_N M}{c^2} \frac{D_{\text{ls}}}{D_{\text{ol}} D_{\text{os}}}}, \end{aligned} \quad (23)$$

$$\begin{aligned} \vartheta_W &\equiv \text{angular radius of the Weyl ring} \\ &\equiv \frac{1}{\sqrt{1+n_W}} \vartheta_E, \end{aligned} \quad (24)$$

$$\begin{aligned} r_E &\equiv \text{radius of the Einstein ring} \\ &\simeq \vartheta_E D_{\text{ol}}, \end{aligned} \quad (25)$$

$$\begin{aligned} r_W &\equiv \text{radius of the Weyl ring} \\ &\simeq \vartheta_W D_{\text{ol}}, \end{aligned} \quad (26)$$

$$n_W \equiv \gamma_W \frac{D_{\text{ls}} D_{\text{ol}}}{D_{\text{os}}}. \quad (27)$$

For a given O–L–S configuration, equation (22) allows us to compute the position of images, or inversely, to infer the position of the light source from the observed position of images. The Weyl lens equation is quadratic, as in general relativity. The corrective factor with respect to general relativity, $1/(1+n_W)$, reduces to 1 when $\gamma_W|_{GR} = 0$ and is greater than 1 for a negative Weyl parameter. Indeed, this lens equation is only valid in the weak field limit (3), leading to n_W always smaller in norm than 1.

Changing the O–L–S distances in the microlensing or gravitational mirage models (17) or (19) would change the Einstein/Weyl radii ((23), (25), (18); (24), (26), (20)), but the radius at which the geodesic potential for photons cancels (1), the interesting closest approach distance (2) and the weak/strong field limiting radii (3), (4) would remain unchanged. Interestingly, the predictions that we derive from equation (22) not only depend on the parameter of the Weyl theory through n_W , but also on the physical properties of the O–L–S system via a combination of the distances D_{ls} , D_{ol} and D_{os} .

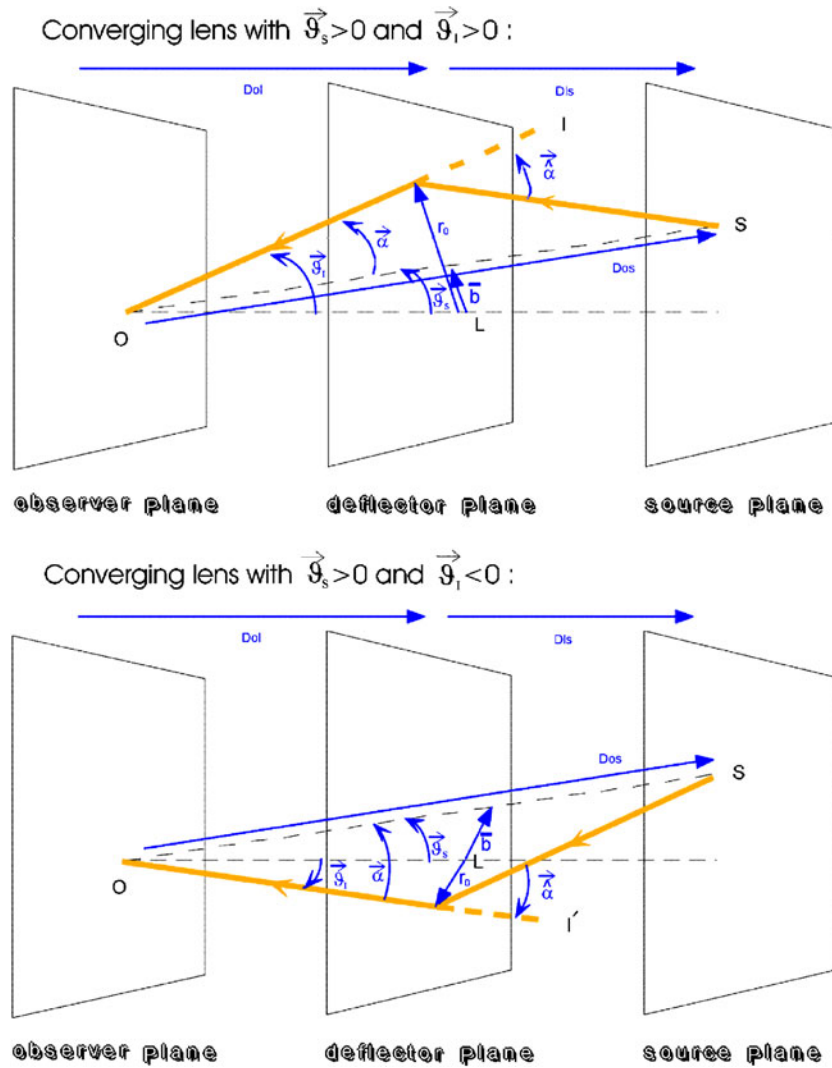


Figure 1. Usual thin lens model of a gravitational mirage where O is the observer, L the gravitational lens, I the image formed, $\vec{\alpha}$ the asymptotic light deflection angle, \vec{b} the impact parameter of the lens on the O–S direction, $\vec{\vartheta}_I$ the angular position of the observed image, with respect to the O–L axis, $\vec{\vartheta}_S$ the angular position of the light source, with respect to the O–L axis. In the case of a diverging lens, there is no image formed from a divergent ray on the opposite side of the source with respect to the O–L axis.

In the case of alignment ($\vartheta_S = 0$) of the observer, the lens and the source, the image is a ring of angular radius $\vartheta_I = \vartheta_W$. The predicted width of the ring ($2 d\vartheta_I$) given as a function of the angular diameter of the light source ($d\vartheta_S$) will be different from the Einsteinian one, even if we refer to the natural size of the physical problem (ϑ_W instead of ϑ_E used in general relativity):

$$d\vartheta_I = \frac{1}{2(1 + n_W)} d\vartheta_S.$$

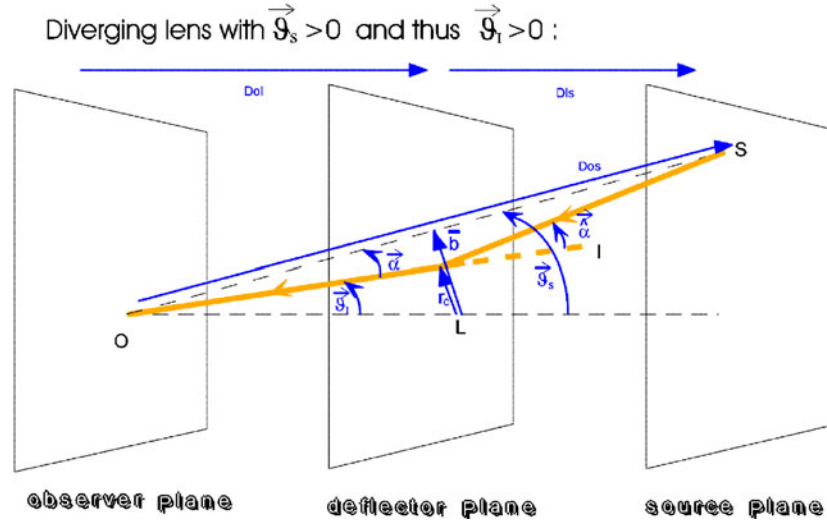


Figure 1. (Continued.)

The Weyl ring will also be thicker than its general relativistic counterpart. Consequently, the amplification, μ , will be larger:

$$\mu \simeq \frac{2\vartheta_W}{(1+n_W) d\vartheta_S},$$

and infinite in the limit of a point-like lens mass, as in general relativity.

In the case of misalignment ($\vartheta_S > 0$), the ring breaks up into two arcs located at

$$\vartheta_{I\pm} = \frac{\vartheta_S \pm \sqrt{\vartheta_S^2 + 4\vartheta_W^2(1+n_W)^2}}{2(1+n_W)},$$

and separated by an angle

$$\Delta\vartheta_I \equiv \vartheta_{I+} - \vartheta_{I-} = \frac{\sqrt{\vartheta_S^2 + 4\vartheta_W^2(1+n_W)^2}}{(1+n_W)}.$$

The sign \pm for ϑ_I distinguishes between images formed on the same side or on the opposite side of the light source with respect to the O–L axis.

The width of the arcs ($2 d\vartheta_{I\pm}$) is given by

$$d\vartheta_{I\pm} = \frac{1}{2(1+n_W)} \left[1 \pm \frac{1}{\sqrt{1 + 4(\vartheta_W/\vartheta_S)^2(1+n_W)^2}} \right] d\vartheta_S.$$

Consequently, each image will be more amplified than in general relativity, with the corresponding amplifications

$$\begin{aligned} \mu_{\pm} &\simeq \frac{1}{4(1+n_W)^2} \left[2 \pm \left((1+n_W) \frac{\Delta\vartheta_I}{\vartheta_S} + \frac{1}{(1+n_W)} \frac{\vartheta_S}{\Delta\vartheta_I} \right) \right] \\ &= \frac{1}{(1+n_W)^2} \frac{\pm [\overline{B}_{\widehat{W}} \pm \sqrt{\overline{B}_{\widehat{W}}^2 + 4}]^2}{4\overline{B}_{\widehat{W}} \sqrt{\overline{B}_{\widehat{W}}^2 + 4}}, \end{aligned}$$

where we define the following dimensionless quantities with respect to \bar{b} , the impact parameter of the deflector in the O–S direction: $\bar{B}_E \equiv \bar{b}/r_E$, dimensionless Einstein impact parameter of the deflector; $\bar{B}_W \equiv \bar{b}/r_W$, dimensionless Weyl impact parameter of the deflector; $\bar{B}_{\hat{W}} \equiv \frac{1}{(1+n_W)}\bar{B}_W$, normalized dimensionless Weyl impact parameter of the deflector.

Hence, the total amplification, when the two images are not resolved, is again larger than the Einsteinian value:

$$\mu_{\text{tot}} \simeq \frac{1}{(1+n_W)^2} \frac{\bar{B}_{\hat{W}}^2 + 2}{\bar{B}_{\hat{W}} \sqrt{\bar{B}_{\hat{W}}^2 + 4}}. \tag{28}$$

3.3.3. *Microlensing.* The above expression and

$$\bar{B}_{\hat{W}}(t) \simeq \bar{B}_{\hat{W}0} \left[1 + \frac{t^2}{T_0^2} \right]$$

are the relevant equations to be used for microlensing amplification curves. T_0 is the time corresponding to the minimal normalized dimensionless Weyl impact parameter of the deflector, $\bar{B}_{\hat{W}0}$.

Strictly speaking, we cannot rescale the amplification curve ($\bar{B}_{\hat{W}} \mapsto \bar{B}_E$) to fit Einsteinian predictions because of the front factor in (28). In view of the upper bound for a negative linear parameter (16) and of the typical distances for microlensing events (17), we find an upper bound for the corrective factor

$$\frac{1}{(1+n_W)} \Big|_{\text{microlens (17)}} - 1 \stackrel{(16)}{\lesssim} 10^{-11}. \tag{29}$$

This corrective factor might be very small for microlensing events if $|\gamma_W|$ happens to be much smaller than our present upper bound on γ_W (16).

The optical depth of microlensing [Na1997] is the probability, at a given time, that a light source is within the corresponding ring radius of a given star lens; hence, it is the probability for this light source to be lensed. To estimate the optical depth in the setting of the Weyl theory, we must integrate over the surface included in the ring radius, thus, over $\pi \vartheta_W^2$. According to (24), the corrective factor (29) enters the integral. One might have hoped that the Weyl theory would substantially increase the optical depth, so maybe account for observations leading to a larger value than initially estimated with general relativity. Unfortunately, owing to the upper bound on $|\gamma_W|$ in (16), the correction is irrelevant with respect to observational uncertainties.

3.3.4. *Gravitational mirages.* For distance scales involved in gravitational mirages, our constraint (16) on a negative Weyl parameter leads to the following maximum deviation from general relativity:

$$\frac{1}{(1+n_W)} \Big|_{\text{mirages (19)}} - 1 \stackrel{(16)}{\lesssim} 10^{-5}. \tag{30}$$

This might be negligible. As gravitational lenses are always convergent for a negative γ_W , the number and parity (position of ϑ_I relative to ϑ_S on the diagram) of the images formed in Weyl gravity are analogous to the Einsteinian case. Consequently, deviations from general relativity are only quantitative, and solely a statistic of gravitational lensing events with different lens mass distribution models would settle the question whether the corrective factor might have observable consequences or not.

3.4. Testing the Mannheim–Kazanas parametrization ($\gamma_W > 0$)

We have argued in section 3.2 that we could not so far use the weak field limit to further constrain a positive γ_W . However, we can investigate the possibility of invalidating the Mannheim–Kazanas theory (9), (12), or try to extract predictions on microlensing or gravitational mirages from this theory where the weak field limit (3) extends to cosmological distances.

3.4.1. Lens equation in the weak field limit. Accordingly, we use the weak field lens equation (22) with the corresponding definitions (23)–(27). Nevertheless, if assuming a positive value of the linear Weyl parameter, additional conditions are necessary when the deflection angle is divergent ($\vec{\alpha} < 0$ for $\vartheta_I > \vartheta_{I0} = \arcsin\left\{\frac{r_{00}}{D_{\text{ol}}}\right\}$, where r_{00} is defined in equation (2)). Those conditions might be inferred from figure 1:

$$\left. \begin{array}{l} \text{sign}(\vec{\vartheta}_I) = \text{sign}(\vec{\vartheta}_S) \\ \vartheta_S \neq 0 \\ \vartheta_S > \vartheta_I \end{array} \right\}. \quad (31)$$

The corrective factor corresponding to the Mannheim–Kazanas parametrization, $1/(1+n_W)$, is still positive but smaller than 1.

3.4.2. Microlensing. As far as microlensing is concerned, the interesting closest approach distance r_{00} (2), at which light deflection becomes divergent, is larger than the microlensing Einstein radius (18) and hence is irrelevant. Thus, light deflection is always convergent on microlensing scales, which means no qualitative deviations from general relativity. Moreover, an estimation of the order of magnitude of the corrective factor for the Mannheim–Kazanas parametrization,

$$1 - \frac{1}{1+n_W} \Big|_{\text{microlens (17)}} \stackrel{(9)}{\simeq} 10^{-5}, \quad (32)$$

shows that microlenses are not an appropriate tool to test the Mannheim–Kazanas theory.

3.4.3. Gravitational mirages. Regarding gravitational mirages, r_{00} (2) is relevant ($\vartheta_{I0} < \vartheta_E$) to the most massive clusters ($M \sim 10^{15} M_{\text{Sun}}$). But in this case, the O and L points of the photon trajectory lie about on the edge of the weak field limit corresponding to the Mannheim–Kazanas parametrization, while $r_0 \sim r_E$ is still well within it.

Nevertheless, let us discuss the corresponding image–position diagram (figure 2) which represents the image position (ϑ_I) solution to the lens equation (22) as the crossing of two curves for a given source position (ϑ_S):

$$F_1(\vec{\vartheta}_I) \equiv \vec{\vartheta}_I - \vec{\vartheta}_S, \quad F_2(\vec{\vartheta}_I) \equiv \text{sign}(\vartheta_I) \left[\left(\frac{1}{1+n_W} - 1 \right) \vec{\vartheta}_S + \frac{\vartheta_W^2}{\vec{\vartheta}_I} \right].$$

The F_2 -curve crosses the ϑ_I -axis at a value ϑ_{I0} corresponding to r_{00} , separating the divergent from the convergent contribution. F_2 presents no foldings. This means that the predictions of the Mannheim–Kazanas theory will be the same as the Einsteinian ones from the point of view of the number of images and their parity, which seems to prevent this theory from being tested using gravitational mirages. One can just say that the Mannheim–Kazanas gravity even needs more dark matter than general relativity (the Weyl radius being smaller than the Einstein

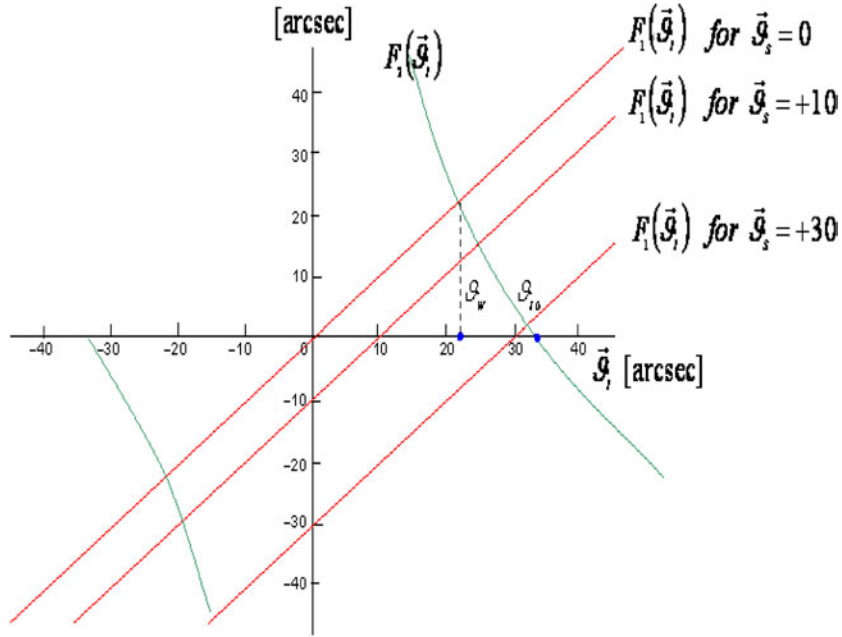


Figure 2. Image-position diagram (F_1 - and F_2 -curves) for a point-like mass lens model ($M_{\text{cluster}} = 10^{15} M_{\text{Sun}}$) in the framework of the Mannheim–Kazanas theory ($\gamma_W \simeq 10^{-26} \text{ m}^{-1}$). The crossings between the F_1 - and F_2 -curves correspond to the image positions (ϑ_I). The case represented here is for $\vec{\vartheta}_S > 0$ and F_1 is plotted at $\vartheta_S = 0$ (alignment), +10 arcsec and +30 arcsec. The values of the variables in the simulation are given in the text.

ring), because the corrective factor is smaller than 1. To illustrate this, note that the simulation discussed (19) provides

$$\frac{1}{1 + n_W} \Big|_{\text{mirages (19)}} \stackrel{(9)}{\simeq} \frac{1}{1 + 0.75 \times 10^{-15+x}} \quad \text{where } M = 10^x M_{\text{Sun}} \quad (33)$$

and clusters with a mass of $\sim 10^{15} M_{\text{Sun}}$ are on the edge of the weak field approximation, as explained.

Note that when $\vartheta_S < \vartheta_{I0}$, the two images formed in the case of misalignment are due to two convergent rays; when $\vartheta_S = \vartheta_{I0}$, one image is from a convergent ray whereas the other one is from a nondeflected ray of light; and when $\vartheta_S > \vartheta_{I0}$, one image is from a convergent ray and the other one is from a divergent ray.

3.5. Attempts in the literature to deduce a negative value for γ_W

In [EdPa1998], Edery and Paranjape are interested in a negative value of the linear Weyl parameter in order to explain gravitational mirages without dark matter. They extract an order of magnitude for γ_W from observations of giant arcs in clusters. It happens to be just the same order of magnitude (the inverse of the Hubble length) as needed in the Mannheim–Kazanas parametrization (9) but with the opposite sign. The idea behind this estimation is to equate the predicted Einstein radius (25) based on an estimated value of the total gravitational mass (luminous plus an *ad hoc* amount of dark matter, to fit observations in the framework of general relativity) with the Weyl radius (26) for the same O–L–S system, but this time with only the luminous matter.

In the cluster lenses used for this purpose, namely A 370, A 2390 and Cl 2244-02, the ratios of the luminous over the total mass were estimated respectively to be $M_L/M_{\text{tot}} \sim 1/200$, $\sim 1/120$ and $< 1/100$, thanks to a complete modelling of each lens in the setting of general relativity [Be *et al* 1990, GrNa1989, Pe *et al* 1991]. However, it is the general relativistic estimate (not a Weyl estimate) of the luminous mass that the authors implicitly inserted into the Weyl radius when using the above luminous to total mass ratios.

Also, in their estimation of the Weyl radii, the authors use the general relativistic relation between the redshift and the distance. In the framework of general relativity, it is the Robertson–Walker metric, solution to the Einstein equations in the presence of matter, which is used to infer the concept of cosmological distance as a function of the redshift, the matter density and the curvature of the universe. However, in the Weyl theory, the Robertson–Walker metric is *not* a solution of the Bach equations with matter! Indeed, the Weyl tensor associated with the Robertson–Walker metric is null, hence the Bach tensor ($B_{\mu\nu}$, equation (2) in [Pi2004]) function of the Weyl tensor is null too, which means that the Bach equations [Bach1921] *in the presence of matter*,

$$B_{\mu\nu} = -1/4\sqrt{\kappa}T_{m\mu\nu},$$

are not satisfied! The concept of distance in the Weyl theory (as well as the concept of time) requires the specification of the conformal factor. This conformal factor is crucial in cosmology to obtain the appropriate Weyl prescription relating observed redshifts to cosmological distances. Otherwise, one can only work in terms of distance ratios (or time ratios, or the mixed time to distance ratios) and angles.

Note that, in the limit of small redshifts, the rough relationship between redshift and distance given in (14) together with the approximation of an Euclidean space ($D_{\text{is}} = D_{\text{os}} - D_{\text{ol}}$) is sufficient to recover the same order of magnitude for γ_W as the one calculated by Edery and Paranjape [EdPa1998]. We obtain as well the natural connection between γ_W and the inverse of the Hubble length:

$$\begin{aligned} |\gamma_W| &= \left(1 - \frac{M_L}{M_{\text{tot}}}\right) \frac{D_{\text{os}}}{D_{\text{is}}D_{\text{ol}}} \\ &\stackrel{\text{(14) and Euclidean approximation}}{\simeq} \left(1 - \frac{M_L}{M_{\text{tot}}}\right) \frac{z_S}{(z_S - z_L)z_L} \frac{H_0}{c} \\ &\stackrel{\text{A370, A2390 and Cl2244-02}}{\simeq} 5 \times 10^{-26} - 6 \times 10^{-26} \text{ m}^{-1}. \end{aligned} \quad (34)$$

Approximation (14) can be obtained simply from the observational redshift which is related to the recession speed of galaxies (v_{recess}) by a simple Doppler effect, and from the empirical Hubble law based on observations:

$$\frac{v_{\text{recess}}}{\text{const } D} \simeq 1 \quad \text{with } \text{const} = H_0|_{\text{exp}}.$$

Interestingly, the Hubble law cited above, and the measured redshift (which is a ratio of frequencies) do not require to specify the conformal factor in the Weyl line element.

Another point concerns the weak field approximation that the authors Edery and Paranjape use, through lens equation (22) and the corresponding definitions of the Weyl radius (26). The gravitational mirages considered in their article,

Lens	z_L	z_S
A 370	0.375	0.724
A 2390	0.231	0.913
Cl 2244-02	0.331	0.83

correspond to observer–lens or to lens–source distances, as can be calculated using the rough estimation (14), that are dangerously close to the weak field limit associated with their estimated value of γ_W . This leaves us dubious about the above result (33).

Finally, in another article [Ed *et al* 2001], Ederly *et al* speak of the ‘theoretical arbitrariness’ of the choice of the conformal factor in the Weyl spherically symmetric solution (equation (3) in [Pi2004]). They furthermore use the conformal transformation given in (13) to argue that the parameter γ_W might be measured as positive for matter particles with $ds_{\text{matter}}^2 \equiv ds^2(r', \gamma'_W > 0)$ in (13); and on the contrary, as negative for photons with $ds_{\text{photons}}^2 \equiv ds^2(r, \gamma_W < 0)$ in (13). Their aim is to try to connect their estimate of γ_W (33) to the Mannheim–Kazanas theory (12) and explain the discrepancy in the sign of the linear Weyl parameter.

However, if the motion of ultra-relativistic particles and photons does not depend on the conformal factor $\chi^2(r)$ and the choice of $\chi^2(r)$ is arbitrary when restricting to those types of particles, we have shown that in contrast, the motion of matter [Pi2004] as well as the definition of distance scales and timescales crucially depends on it. Because our world is made of matter, it is not conformally invariant (we indeed have clocks and rods to make measurements). Hence, the conformal factor is not arbitrary, as long as we are looking for a theory to describe Nature. Nature has chosen a specific conformal factor. The Weyl theory should even be called ‘Weyl theories’, because it in fact corresponds to a class of theories, each theory being specified by a choice of $\chi^2(r)$ and (γ_W, k_W) while $\beta_W \equiv \frac{G_N M}{c^2}$. The Mannheim–Kazanas theory is a particular example.

Moreover, even though the linear γ_W -contribution to the effective potential might have the opposite effect on massless in comparison with massive particles (section 3.1 in [Pi2004]), it is necessary to use the same radial variable in order to have the same definition of γ_W when comparing estimates of γ_W obtained from photon trajectories with those obtained from massive particle motion.

4. Conclusions

Regarding the change of the apparent position of stars, the weak field regime applied to the VLBI data gave an upper bound on the linear parameter: $|\gamma_W| \sim 10^{-18} \text{ m}^{-1}$ (8) that was improved with the use of the recent Cassini Doppler data, to lead to $|\gamma_W| \sim 10^{-19} \text{ m}^{-1}$ (11). However, solar system experiments do not settle the sign of γ_W . A positive γ_W decreases the light deflection angle, while a negative one increases it with respect to general relativity. A negative parameter might thus be an alternative to too large an amount of dark matter. Analysing the expression for the asymptotic deflection angle in the Weyl theory, it was found to allow for a diverging effect at closest approach distances larger than r_{00} (2) function of the mass of the lens, *if γ_W is positive*. However, the convergent-to-divergent transition is not explicitly relevant to solar system experiments.

We considered, when applicable, the weak field approximation to study microlensing and gravitational mirages. The strong field limit was found to be useless for our purposes; because we only have in hand an upper bound on $|\gamma_W|$. The lower bound given by general relativity ($\gamma_W = 0$) leads to a strong field limit only valid at infinity.

For a negative γ_W , the condition for unbound photon orbits derived in the preceding article [Pi2004] and the existence of gravitational mirages were used to improve constraints on the γ_W -parameter obtained from solar system data. The upper bound on the absolute value of γ_W was accordingly lowered to about 10^{-31} m^{-1} . The characteristics of the microlensing or gravitational mirage curve in the Weyl theory cannot, by a simple rescaling of the mass or the ring radius, be recast into the Einsteinian predictions. However, the corrective factor,

$1/(1+n_W)$, function of γ_W and the O–L–S distances, is small. Indeed, it is equal to 1 when γ_W is equal to zero (general relativity), and differs from its Einsteinian value with a maximum of $\sim+10^{-11}$ for microlenses, or $\sim+10^{-5}$ for gravitational mirages. This means that it is negligible for microlenses and might effectively be negligible for gravitational mirages too. The latter point requires further study. Our estimate of the corrective factor is based upon our upper bound on $|\gamma_W|$, for a negative Weyl parameter and a point mass lens model.

In the Mannheim–Kazanas theory ($\gamma_W \simeq 10^{-26} \text{ m}^{-1} > 0$), gravitational mirages do not seem to be an appropriate test. Indeed, the predictions will be the same as the general relativistic ones, from the point of view of the number of images and of their parity. One can just say that the Mannheim–Kazanas gravity needs even more dark matter than general relativity, because the Weyl radius is smaller than the Einstein one.

In microlensing, the interesting closest approach distance separating the convergent from the divergent contributions (r_{00}) is cosmological in the Mannheim–Kazanas theory, and thus irrelevant. Moreover, the smallness of the corrective factor $\sim(1-10^{-5})$ showed that one cannot distinguish between general relativity and the Mannheim–Kazanas theory from the point of view of microlensing, either.

We commented on the concept of distance that needs to be defined consistently in the Weyl theory. Also, to be consistent, the weak (or strong) field limit on the radial distance, measured from the gravitational deflector, has to be verified on the photon path, all the way from the light source to the observer. Thus, the limit does not only apply to the lens–observer and lens–source distances, but also to the closest approach distance of the photon onto the lens.

Even though the bounds we obtained on γ_W are conservative in comparison with Mannheim’s estimated value of $\gamma_W \sim +10^{-26} \text{ m}^{-1}$, they are preferable because they are not biased by any arbitrary assumption on the conformal factor. Moreover, the particular value of Mannheim and Kazanas belongs to the allowed range that we derived.

Acknowledgments

The research work presented here was carried out in the FYMA Institute at the University of Louvain la Neuve during a PhD thesis financed by the IISN research assistantship. I am grateful to Professor M Festou (LAT, Observatoire Midi-Pyrénées, Toulouse) for pertinent advice.

References

- [Bach1921] Bach R 1921 Zur Weylschen Relativitätstheorie und der Weylschen Erweiterung des Krümmungstensorbegriffs *Math. Z.* **9** 110–35
- [Be et al 1990] Bergmann A G, Petrosian V and Lynds R 1990 Gravitational lens models of arcs in clusters *Astrophys. J.* **350** 23–35
- [Be et al 2003] Bertotti B, Iess L and Tortora P 2003 A test of general relativity using radio links with the Cassini spacecraft *Lett. Nature* **425** 374–6
- [Co et al 1974] Counselman C C III, Kent S M, Knight C A, Shapiro I I, Clark T A, Hinteregger H F, Rogers A E E and Whitney A R 1974 Solar gravitational deflection of radio waves measured by Very-Long-Baseline Interferometry *Phys. Rev. Lett.* **33** 1621–3
- [EdPa1998] Edery A and Paranjape M B 1998 Classical tests for Weyl gravity: deflection of light and time delay *Phys. Rev. D* **58** 1–8 (*Preprint astro-ph/9708233*)
- [Ed et al 2001] Edery A, Méthot A A and Paranjape M B 2001 Gauge choice and geodetic deflection in conformal gravity *Gen. Rel. Grav.* **33** 2075–9 (*Preprint astro-ph/0006173*)

- [Eu *et al* 1999] Eubanks T M, Martin J O, Archinal B A, Josties F J, Klioner S A, Shapiro S and Shapiro I I 1999 Advances in Solar System tests of gravity *Preprint* online at <ftp://casa.usno.navy.mil/navnet/postscript/prd-15.ps>
- [FoSr1976] Fomalont E B and Sramek R A 1976 Measurements of the Solar gravitational deflection of radio waves in agreement with general relativity *Phys. Rev. Lett.* **36** 1475–8
- [GAIA2000] GAIA Science Advisory Group 2000 GAIA Composition and formation evolution of the galaxy. Results of the concept and technology study Online at <http://astro.estec.esa.nl/SA-general/Projects/GAIA/>, march-report. pdf, version 1.6
- [GrNa1989] Grossman S A and Narayan R 1989 Gravitationally lensed images in ABDEL 370 *Astrophys. J.* **344** 637–44
- [Le *et al* 1995] Lebach D E, Corey B E, Shapiro I I, Ratner M I, Webber J C, Rogers A E E, Davis J L and Herring T A 1995 Measurements of the solar gravitational deflection of radio waves using Very Long Baseline Interferometry *Phys. Rev. Lett.* **75** 1439–42
- [Na1997] Narayan R 1997 *Lectures on Gravitational Lensing* Harvard-Smithsonian Center for Astrophysics (*Preprint* astro-ph/9606001 v2)
- [Ma1994] Mannheim P D 1994 Microlensing, Newton–Einstein gravity, and conformal gravity *UCONN-94-10 Preprint* astro-ph/9412007
- [Ma1995] Mannheim P D 1995 Conformal cosmology and the age of the universe *UCONN* 95-08
- [MaKa1989] Mannheim P D and Kazanas D 1989 Exact vacuum solution to conformal Weyl gravity and galactic rotation curves *Astrophys. J.* **342** 635–8
- [Pe *et al* 1991] Pello R, Le Borgne J-F, Soucail G and Mellier Y 1991 A straight gravitational image in ABDEL 2390: a striking case of lensing by a cluster of galaxies *Astrophys. J.* **366** 405–11
- [Pi1997] Pireaux S 1997 Etude de solutions particulières d’une théorie invariante conforme de la gravitation *Graduate Thesis* Université catholique de Louvain (UCL)
- [Pi2002] Pireaux S 2002 Light deflection experiments as a test of relativistic theories of gravitation *PhD Thesis* Université catholique de Louvain (UCL)
- [Pi2004] Pireaux S 2004 Light deflection in Weyl gravity: critical distances for photon paths *Class. Quantum Grav.* **21** 1897–913 (*Preprint* gr-qc/0403071)
- [RoCa1984] Robertson D S and Carter W E 1984 Relativistic deflection of radio signals in the solar gravitational field measured with VLBI *Nature* **310** 572–4
- [Ro *et al* 1991a] Robertson D S, Carter W E and Dillinger W H 1991 New measurements of solar gravitational deflection of radio signals using VLBI *Nature* **349** 768–70
- [Ro *et al* 1991b] Robertson D S, Carter W E and Dillinger W H 1991 A New measurement of solar gravitational deflection of radio signals using VLBI *Proc. AGU Chapman Conf.: ‘Geodetic VLBI: Monitoring Global Change’ (Washington, DC Apr. 1991)* pp 22–6, 203–12
- [Se *et al* 1970] Seielstad G A, Sramek R A and Weiler K 1970 Measurement of the deflection of 9.602-GHz radiation from 3C279 in the solar gravitational field *Phys. Rev. Lett.* **24** 1373–6
- [Sh *et al* 2004] Shapiro S S, Davis J L, Lebach D E and Gregory J S 2004 Measurement of the solar gravitational deflection of radio waves using geodetic Very-Long-Baseline Interferometry data 1979–1999 *Phys. Rev. Lett.* **92** 121101
- [Turn1999] Turner M S 1999 Dark matter, dark energy and fundamental physics *Proc. ‘Physics in Collision’ (Ann Arbor, MI, 24–26 June 1999)* ed M Campbell and T M Wells (Hackensack, NJ: World Scientific) (*Preprint* astro-ph/9912211)
- [Tury *et al* 2004] Turyshv S G, Shao M and Nordtvedt K Jr 2004 The laser astrometric test of relativity (LATOR) mission *Astron. Nachr.* **325** 267–77 (*Preprint* gr-qc/0311020 v1, gr-qc/0311049 v1)
- [Wi2001] Will C M 2001 The confrontation between general relativity and experiments *Living Rev.* **4** Online at www.livingreviews.org/Articles/Volume4/2001-4will