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Time scales in LISA

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Abstract
The LISA mission is a space interferometer aiming at the detection of gravitational waves in the $[10^{-4}, 10^{-1}]$ Hz frequency band. In order to reach the gravitational wave detection level, a time delay interferometry (TDI) method must be applied to get rid of (most of) the laser frequency noise and optical bench noise. This TDI analysis is carried out in terms of the coordinate time corresponding to the Barycentric Coordinate Reference System (BCRS), TCB, whereas the data at each of the three LISA stations are recorded in terms of each station proper time. We provide here the required proper time versus BCRS time transformation. We show that the difference in rate of station proper time versus TCB is of the order of 5×10^{-8}. The difference between station proper times and TCB exhibits an oscillatory trend with a maximum amplitude of about 10^{-3} s.

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(Some figures in this article are in colour only in the electronic version)

1. Introduction

The LISA mission [7] is a space interferometer aiming at the detection of gravitational waves (GW) in the $[\sim 10^{-4}, \sim 10^{-1}]$ Hz frequency band. Gravitational waves crossing the LISA quasi-equilateral triangular constellation are detected through the induced change in the station inter-distances. The latter also depend on time due to the gravitational field of the Sun [2], mostly, and planets, what we call a 'geometry (G) effect'.

In order to reach the gravitational wave detection level, a time delay interferometry (TDI) method (see [15] for a review and references therein) must be applied to get rid of (most of) the laser frequency (LF) noise and optical bench (OB) noise. In other words, TDI is needed to bring those LF plus OB (physically indistinguishable) noises down to the level of the other noises: quantum (Q), fibre (F), residual proof mass motion (PM) noises . . .. The TDI method consists in combining numerically data fluxes at the stations (rather than combining the laser beams physically) with an appropriate delay. Hence, the so-called TDI observables are closed loop combinations of the different laser links with appropriate delays (combination of photon-flight time t_{ij} between two stations $i \to j$ which correspond to station inter-distances) that cancel (almost all) the laser frequency noise and optical bench noise.

The TDI analysis is carried out in terms of the coordinate time t corresponding to the Barycentric Coordinate Reference System (BCRS), the so-called TCB, whereas the data at each of the three LISA stations, $k = 1, 2, 3$, are archived in terms of the station proper time τ_k.

We modelled the orbitography of the three stations ($\vec{x_i}$, $\vec{v_i} \equiv \mathrm{d}\vec{x_i}/\mathrm{d}t$ with $t = \mathrm{TCB}$ and $i = 1, 2, 3$) *classically* [6] in the gravitational field of the sole Sun, and the laser links, that is the photon-flight time t_{ij}, *relativistically* [2] as a function of the position and velocities of stations i and j at emission time.

We here provide the corresponding analytical proper time versus TCB transformation required to apply the TDI analysis. We show that the difference between station proper time and TCB reaches about 0.5 s over a one-year mission and exhibits an oscillatory trend with a maximal amplitude of 0.0014 s. The difference in rate of station proper time versus TCB is of the order of 1.5×10^{-8}.

We then show how this proper time versus TCB transformation fits in a general relativistic TDI analysis.

2. Proper time versus coordinate time transformation

2.1. Classical orbitography

We assume a classical orbitography for the three LISA spacecraft $k = 1, 2, 3$, in the BCRS, as given in [6]. Those BCRS coordinates (x_k, y_k, z_k), for arbitrary initial conditions, can be rewritten in terms of rotated Keplerian ellipses $(x_{\mathrm{ell}\,k}, y_{\mathrm{ell}\,k}, z_{\mathrm{ell}\,k})$ as

$$\begin{pmatrix} x_k \\ y_k \\ z_k \end{pmatrix} = \Re^{-1} \begin{pmatrix} x_{\mathrm{ell}\,k} \\ y_{\mathrm{ell}\,k} \\ z_{\mathrm{ell}\,k} \end{pmatrix} \qquad (1)$$

with

$$\Re^{-1} \equiv \begin{pmatrix} +\cos\Omega_k \cos\omega - \sin\Omega_k \sin\omega \cos \mathrm{i} & -\cos\Omega_k \sin\omega - \sin\Omega_k \cos\omega \cos\mathrm{i} & +\sin\Omega_k \sin\mathrm{i} \\ +\sin\Omega_k \cos\omega + \cos\Omega_k \sin\omega \cos\mathrm{i} & -\sin\Omega_k \sin\omega + \cos\Omega_k \cos\omega \cos\mathrm{i} & -\cos\Omega_k \sin\mathrm{i} \\ +\sin\omega \sin\mathrm{i} & +\cos\omega \sin\mathrm{i} & +\cos\mathrm{i} \end{pmatrix}$$

and

$$\begin{pmatrix} x_{\mathrm{ell}\,k} \\ y_{\mathrm{ell}\,k} \\ z_{\mathrm{ell}\,k} \end{pmatrix} \equiv \begin{pmatrix} a(\cos\Psi_k - e) \\ a\sqrt{1-e^2}\sin\Psi_k \\ 0 \end{pmatrix}$$

where

$$a = 1\mathrm{AU}, \qquad e = \sqrt{1 + \frac{4}{\sqrt{3}}\frac{L}{2a}\cos\nu + \frac{4}{3}\left(\frac{L}{2a}\right)^2} - 1, \qquad i = \mathrm{arctg}\left(\frac{\frac{L}{2a}\sin\nu}{\sqrt{3/2} + \frac{L}{2a}\cos\nu}\right)$$

and ω are the common semi-major axis, eccentricity, inclination and argument of the periaster of the three spacecraft orbits, respectively. The optimal inclination of the LISA triangle on the ecliptic is $\nu = \frac{\pi}{3} + \frac{5}{8}\frac{L}{2a}$ [10] with $L = 5 \times 10^9$ m, the average interferometric arm-length. The longitude of the ascending node, Ω_k, is particular to a given spacecraft k and is given in terms of that of the first one with a phase shift θ_k:

$$\Omega_k = \Omega_1 - \theta_k \qquad \text{with} \qquad \theta_k \equiv -2(k-1)\frac{\pi}{3}.$$

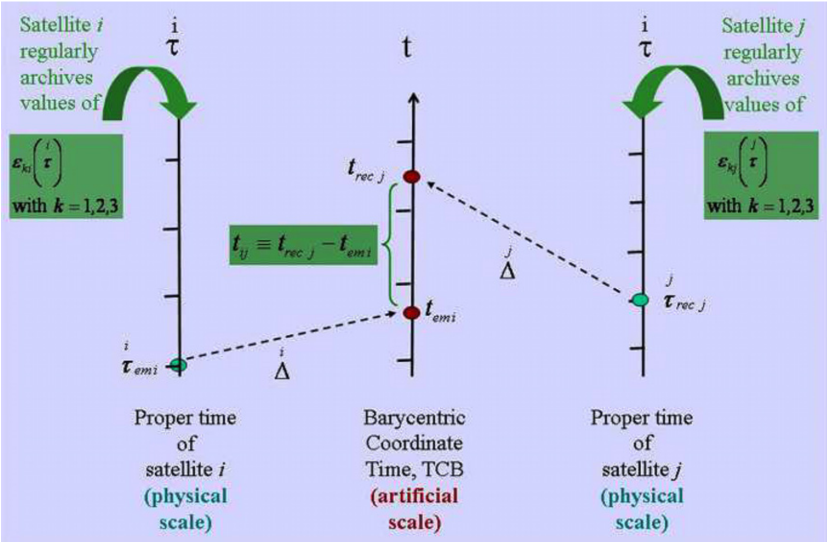

Figure 1. The TDI method is developed in terms of the coordinate time scale, t, associated with the Barycentric Coordinate Reference System which is an artificial time scale (it can only be computed, not measured). Since each spacecraft i or j has its own physical time scale in terms of the proper time beaten by its clock $\overset{i}{\tau}$ or $\overset{j}{\tau}$, a coordinate time t is the natural 'common language'. However, events (such as emission or reception of a signal) and proper frequency shifts are recorded locally at each spacecraft on their own proper time scale. Hence, a time transformation, $\overset{i}{\Delta}$ or $\overset{j}{\Delta}$, is required with $\overset{k}{\Delta} \equiv \overset{k}{\tau} - t$.

The time parametrization of the orbits is given by the equation of the eccentric anomaly Ψ_k of each spacecraft,

$$\Psi_k - e \sin \Psi_k = M_k, \tag{2}$$

with the mean anomaly

$$M_k = \frac{2\pi}{T}(t - t_0) + M_{k0}$$

in terms of the orbital period, T (provided by Kepler's third law, $2\pi/T = \sqrt{GM/a^3}$), and the mean anomaly of spacecraft k at initial time t_0, that is $M_{k0} \equiv M_k(t = t_0)$. Mean anomalies are related to that of the first spacecraft through the phase shift:

$$M_k = M_1 + \theta_k.$$

The common spacecraft orbit eccentricity being small, the eccentric anomaly equation (2) can be developed at first order (with respect to e):

$$\Psi_k \simeq +\frac{2\pi}{T}(t - t_0) + M_{10} + \theta_k + e \sin\left(\frac{2\pi}{T}(t - t_0) + M_{10} + \theta_k\right). \tag{3}$$

BCRS position and eccentric anomaly equations used in [2] correspond to particular initial conditions ($t_0 = 0$, $\omega = 3\pi/2$, $\Omega_1 = 3\pi/2$, $M_{10} = 0$) without any planets (which means that both t_0 and M_{10} are completely arbitrary). When planets are modelled, the LISA guiding centre, that is the projection of LISA's centre of mass on the ecliptic plane, is supposed to be about 20° behind the Earth at initial time t_0. This means that t_0 and M_{10} are no longer arbitrary. For example, if the initial mission time is $t_0 = 0$ on epoch 1 January 2012 at 00:00:00, we find $M_{10} \simeq -0.955002$ from Keplerian Earth ephemeris for $\omega = 3\pi/2$, $\Omega_1 = 3\pi/2$.

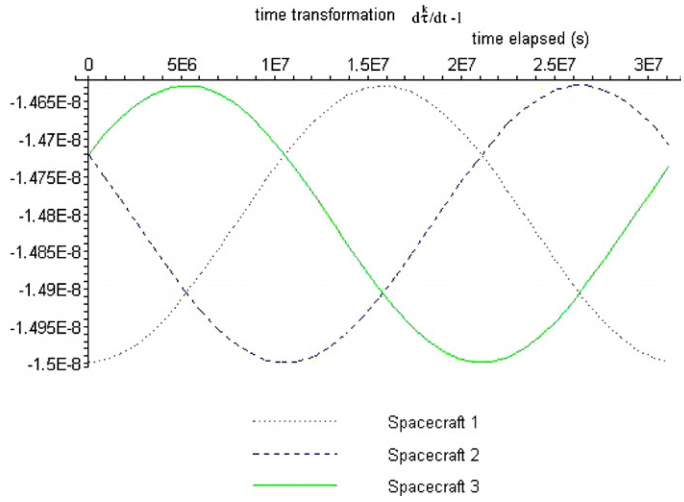

Figure 2. Differential proper versus TCB time transformation for spacecraft k (equation (4)).

2.2. Relativistic time transformation

We now wish to compute the time transformation between proper time, $\overset{k}{\tau}$, of the clock on board spacecraft k, as a function of coordinate (BCRS) time t, called TCB; since coordinate time is the common 'language' between the different spacecrafts ($k = 1, 2, 3$) and the time used in the TDI method.

If we consider only the gravitational field due to the Sun, the time transformation is given by (figure 2)

$$\mathrm{d}s^2 = c^2\,\mathrm{d}\overset{k}{\tau}{}^2 \simeq \left(1 - 2\frac{w_k}{c^2} - \frac{v_k^2}{c^2}\right)c^2\,\mathrm{d}t^2$$

$$\Rightarrow \mathrm{d}\overset{k}{\tau} \simeq \left(1 - \frac{w_k}{c^2} - \frac{v_k^2}{2c^2}\right)\mathrm{d}t \tag{4}$$

where c is the speed of light in vacuum, $w_k \equiv \frac{GM}{r_k}$ with G being Newton's gravitational constant, M the mass of the Sun, r_k the radial distance relative to the Sun at time t and v_k the velocity of spacecraft k at time t in the Barycentric Coordinate Reference System. We can compute the norm of the satellite Keplerian velocity and Keplerian radial distance using equations (1), leading to

$$v_k^2 = \left(\frac{2\pi}{T}\right)^2 a^2 \frac{1 + e\cos\Psi_k}{1 - e\cos\Psi_k}. \tag{5}$$

$$r_k = a(1 - e\cos\Psi_k). \tag{6}$$

Substituting those expressions in the time transformation (4), we obtain

$$\begin{aligned}\overset{k}{\Delta} \equiv \overset{k}{\tau} - t &\simeq -\frac{GM}{c^2 a}\int \frac{1}{1 - e\cos\Psi_k}\,\mathrm{d}t - \frac{a^2}{2c^2}\left(\frac{2\pi}{T}\right)^2 \int \frac{1 + e\cos\Psi_k}{1 - e\cos\Psi_k}\,\mathrm{d}t \\ &\simeq -\frac{GM}{c^2 a}\frac{T}{2\pi}\Psi_k - \frac{a^2}{2c^2}\frac{2\pi}{T}\Psi_k - \frac{a^2}{2c^2}\frac{2\pi}{T} e\sin\Psi_k + cst \\ &\simeq \overset{k}{\tau}_0 - t_0 - \frac{\sqrt{GMa}}{2c^2}\left[3(\Psi_k - \Psi_{k0}) + e(\sin\Psi_k - \sin\Psi_{k0})\right].\end{aligned} \tag{7}$$

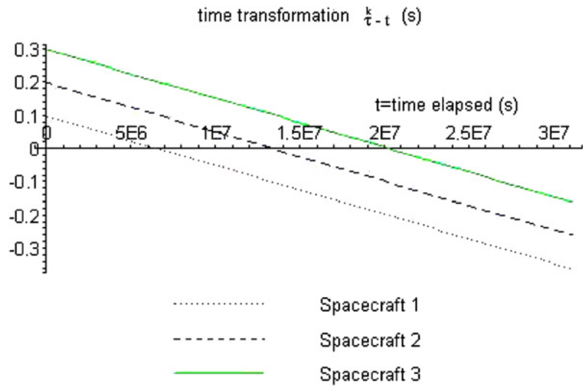

Figure 3. Proper versus TCB time transformation (equation (7)) for spacecraft $k = 1, 2, 3$, integrated over a one-year mission. In this figure, for the sake of illustration, there is an initial shift of 0.1 ; 0.2; 0.3 s for the clock aboard spacecraft $k = 1, 2$ or 3, respectively.

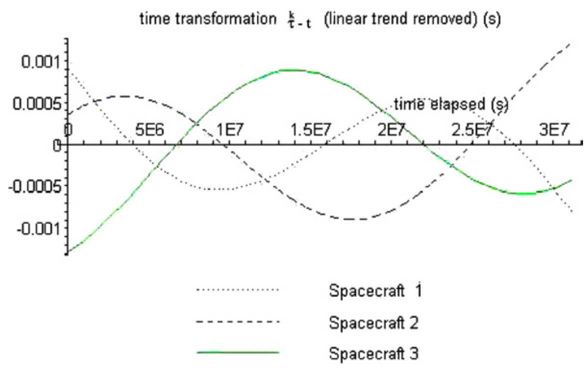

Figure 4. Differential proper versus TCB time transformation (equation (7)) for spacecraft $k = 1, 2, 3$, the linear trend is removed using a least-squares fit method.

The integration constant is given by $\overset{k}{\tau} = \overset{k}{\tau}_0$ and $\Psi_k = \Psi_k(t_0) \equiv \Psi_{k0}$ at initial coordinate time $t = t_0$. The integration was performed through the change in variable $\mathrm{d}t = T\,(1 - e\cos\Psi_k)\,/2\pi\,\mathrm{d}\Psi_k$, using the time derivative of the exact expression for the eccentric anomaly (2), and Kepler's third law. From the implicit equation (2) providing $\Psi_k(t)$, one can compute $\overset{k}{\Delta}$ as a function of time t (figures 3 and 4). Alternatively, one could use the first-order expression (3).

To obtain the figures in section 6, the initial time was chosen as $t_0 = 0$, the initial offsets of the clocks where $\overset{1}{\tau}_0 = 0.1$ s, $\overset{2}{\tau}_0 = 0.2$ s, $\overset{3}{\tau}_0 = 0.3$ s with initial conditions $t_0 = 0, \omega = 3\pi/2, \Omega_1 = 3\pi/2$ and $M_{10} = 0$. The cumulative $\overset{k}{\tau} - t$ effect reaches about half a second over one year. In figure 4, the linear trend is removed. The amplitude of the $\overset{k}{\Delta}$ oscillatory behaviour is $\sim 10^{-3}$ s.

3. LISA model and TDI technique: a coherent general relativistic approach

We now recall how this proper time of spacecraft k versus TCB transformation, $\overset{k}{\Delta}$, fits in the TDI picture. Let us consider data flow variables ij, collected in a vector,

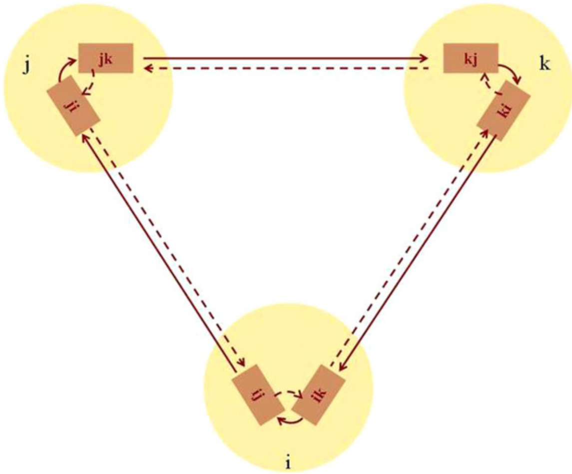

Figure 5. Model for the LISA mission: double laser links between spacekraft i, j, k and double fibre links between the two lasers aboard each spacecraft.

$$\overrightarrow{\varepsilon} \equiv (\varepsilon_{31}, \varepsilon_{12}, \varepsilon_{23}; \varepsilon_{21}, \varepsilon_{32}, \varepsilon_{13}; \varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33}). \tag{8}$$

We briefly describe the content of the above vector[1] using figure 5. Each fibre data-flow variable,

$$\varepsilon_{jj}(t) \equiv (\Gamma^{ijk}(t) - \Gamma^{kji}(t))/2 \qquad \text{with} \quad j = 1, 2, 3, \tag{9}$$

consists of an antisymmetric combination of the two data flows at station j,

$$\Gamma^{ijk}(t) \equiv \Gamma^{ijk}|_{t,\ \text{at}\ j\ \text{with LF,OB,F,PM noises}} \tag{10}$$

and $\Gamma^{kji}(t)$, that compare the frequencies of the two local lasers (jk and ji) via fibre links. Each laser-link data-flow variable,

$$\varepsilon_{ij}(t) \equiv E_{ij}\Lambda^{ki}(t) - \Lambda^{ij}(t) \qquad \text{with} \qquad i \neq j, j \neq k, k \neq i \text{ and } i, j, k = 1, 2, 3, \tag{11}$$

consists of comparing, at station j, the frequency shift between the incoming laser beam $i \to j$ and the local laser aligned in direction $j \to i$ at station j,

$$\Lambda^{ij}(t) \equiv \Lambda^{ij}|_{t,\ \text{at}\ j\ \text{with LF,Q,OB,PM noises + GW signal + G}}, \tag{12}$$

with that at station i, $\Lambda^{ki}(t)$, considering that the light has travelled a path $i \to j$ which translates into a corresponding time delay t_{ij}. In terms of operators, it means dealing with the time-delay operator E_{ij} so that, for a given function of time $f(t)$,

$$E_{ij}(f(t)) \equiv f(t - t_{ij}). \tag{13}$$

The (coordinate-)time transfer between spacecraft i and spacecraft j,

$$t_{ij} \equiv t_{\text{rec}\ j} - t_{\text{em}\ i}, \tag{14}$$

is the barycentric coordinate flight-time of photons between spacecraft i (photon emitted at time $t_{\text{em}\,i}$ by spacecraft i) and spacecraft j (photon received at time $t_{\text{rec}\ j}$ by spacecraft j). This

[1] Our notations $(\varepsilon_{31}, \varepsilon_{12}, \varepsilon_{23}; \varepsilon_{21}, \varepsilon_{32}, \varepsilon_{13}; \varepsilon_{11}, \varepsilon_{22}, \varepsilon_{33})$ correspond to $(U_1, U_2, U_3; -V_1, -V_2, -V_3; Z_1, Z_2, Z_3)$ in [8].

quantity is useful in TDI techniques: data-flow variables are combined with appropriate time delays to form TDI observables which are (almost) free of LF and OB noises:

$$\text{TDI observable}_l = \sum_{m=1}^{m_{\max}} d_l^m \varepsilon^m \tag{15}$$

where

$$\vec{d}_l = m_{\max} - \text{uple polynomial of } p \text{ variables } E_{ij}(t) \tag{16}$$

is the lth TDI generator of a chosen generating set with $l = 1, \ldots, l_{\max}$. The variable $l_{\max}$ is the total number of generators in the chosen generating set and $m_{\max}$ is the number of data-flow variables considered in the vector $\vec{\varepsilon}$. The size, n with $n \leqslant l_{\max}$, of the smallest generating set of polynomials that cancel LF and OB noises, as well as $m_{\max}$ or p, depends on the level of modelling of the LISA mission.

Indeed, one distinguishes between 1st-, 1.5th- and 2nd-generation TDI. The first-generation TDI assumes constant (in time) and symmetric time delays ($t_{ij} = t_{ji}$). This leads to $p = 3$ time-delay operators, a minimum of $n = 4$ TDI generators and $m_{\max} = 6$ data-flow variables (or 9 if LISA's internal motions are considered). The set of first-generation-TDI observables forms the first module of syzygies over a ring of $p = 3$ variables [5]. For ideal, perfect, identical (stable, accurate, without shift nor drift) clocks beating the time t aboard the three stations, if the time delays t_{ij} are known exactly, within the first-generation-TDI assumptions, the LF (and OB) noises are exactly cancelled in first-generation-TDI observables formed from the data-flow variables recorded at each spacecraft.

The 1.5th-generation TDI still assumes constant time delays, but they are not reciprocal anymore ($t_{ij} \neq t_{ji}$). Hence, the number of time-delay operators is doubled $p = 6$, while $n = 6$ and $m_{\max} = 9$. The 1.5th-generation-TDI observables still form a module over a ring of $p = 6$ variables [8]. Hence, within the 1.5th-generation-TDI assumptions, for ideal perfect identical clocks beating the time t and exact knowledge of t_{ij}, the LF and OB noises are still exactly cancelled in 1.5th-generation-TDI observables.

The second-generation TDI relaxes the constant time-delay assumption ($t_{ij} \equiv t_{ij}(t)$) with respect to the 1.5th-generation TDI. Consequently, the time-delay operators ($p = 6$) do not commute anymore. A new module still must be found. In [13], the authors propose six new, more complex, generalized Michelson- and Sagnac-type second-generation-TDI observables. For those, the order in which the delays are applied matters. Those do not remove *exactly* the LF and OB noises, even for ideal perfect identical clocks beating the time t and exact knowledge of t_{ij}, but those bring them down to an acceptable level for LISA.

Let us first consider the different TDI generations with respect to LISA geometry (orbitography and laser links) modelling. The first-generation-TDI assumptions are only met by a rigid, motion-less (this implies that no gravitational bodies are around to cause any motion) LISA constellation model. At this level of modelling, each time delay is given by the corresponding constant interferometric arm-length, slightly different from the generic arm-length L: $t_{ij} = L_{ij}/c$ with $L_{ij} \equiv \vec{x}_j - \vec{x}_i$.

The 1.5-generation-TDI assumptions allow for a rotating (around its centre of mass and around the barycentre) rigid LISA. At that stage, the Sagnac [9] and aberration effects cause the non-reciprocity of time-delays [3]. Indeed, for a same path-length, light rays travelling clockwise and counterclockwise do not take the same time (Sagnac effect). Additionally, there is a motion of the arm-length ij with respect to the barycentre causing an aberration effect. The model for LISA corresponding to 1.5-generation-TDI assumptions consists in *classical* motion (Keplerian) of the three stations in the gravitational field of the Sun up to first order in eccentricity ($e_{\text{LISA}} \cong 0.0096$).

The true LISA requires second-generation TDI. A *classical* Keplerian orbital motion around the Sun, if higher orders in eccentricity are considered, causes a flexing of the constellation: $L_{ij}(t)$ is a simple periodic function of t if only the Sun is considered, the so-called breathing of the triangle. If the Newtonian perturbation of planets is considered, the flexing becomes more complex. Moreover, if a gravitational *relativistic* description of photon time transfer is adopted, t_{ij} has additional time-dependant contributions due to gravitational relativistic effects. Indeed, in [2, section III], a native, coherent, gravitational relativistic description of the laser link provides $t_{ij} = \overset{(0)}{t}_{ij} + \overset{(1/2)}{t}_{ij} + \overset{(1)}{t}_{ij}$, as a function of the positions and velocities of the emitting and receiving spacecraft at emission time. The 0th order in $GM/c^2 \sim v^2/c^2$, $\overset{(0)}{t}_{ij}$, is the classical time taken by light to travel $L_{ij}(t)$ at velocity c, the $1/2$th order contains the Sagnac and aberration effects and the 1st order, light deflection or the so-called Shapiro delay. In [2, section VI], these $\overset{(0)}{t}_{ij}$, $\overset{(1/2)}{t}$ and $\overset{(1)}{t}_{ij}$ contributions to the photon flight time were evaluated numerically to 5×10^9 m$/c \approx 16.7$ s plus a flexing of amplitude of 48 000 km$/c \approx 0.17$ s, $\sim 3 \times 10^{-3}$ s and $< 10^{-7}$ s respectively, assuming *classical* ephemerides of the three LISA stations. In [12], *relativistic* ephemerides of the three LISA stations are provided. The authors compute numerically $L_{ij\,\text{relativistic}}(t) - L_{ij\,\text{classical}}(t)$ and show that it reaches up to about 3 km over a year, corresponding to an extra $\sim 10^{-5}$ s in $\overset{(0)}{t}_{ij}$. A realistic model of LISA's orbital motion also leads to geometric frequency shifts in expression (12). In [2, section V], the native, coherent, gravitational relativistic description of the laser link also provides $\Lambda_G^{ij} = \overset{(1/2)}{\Lambda}{}^{ij}_{G} + \overset{(1)}{\Lambda}{}^{ij}_{G} + \overset{(3/2)}{\Lambda}{}^{ij}_{G}$, as a function of the positions and velocities of the emitting and receiving spacecraft at emission time. The 1/2th order is the Doppler shift due to the relative velocities of the emitting and receiving stations in the special relativistic framework; it is reciprocal. Higher order terms that contain gravitational relativistic effects such as the Einstein gravitational Doppler shift in $\overset{(1)}{\Lambda}{}^{ij}_{G}$, are non-reciprocal. In [2, section VI], these contributions to photon frequency shift were evaluated numerically assuming *classical* ephemerides of the three LISA stations, leading to $\overset{(1/2)}{\Lambda}{}^{ij}_{G} \sim 7 \times 10^{-8}$, $\overset{(1)}{\Lambda}{}^{ij}_{G} \sim 2 \times 10^{-13}$ and $\overset{(3/2)}{\Lambda}{}^{ij}_{G} \sim 2 \times 10^{-14}$. Hence, the geometric frequency shift is irrelevant to LISA, owing to the mission's detection frequency bandwidth.

Let us now consider the TDI analysis from the point of view of LISA time scales. TDI-observables (1st, 1.5th and future module for 2nd generation) should cancel exactly the LF and OB noises for ideal, perfect, identical clocks beating the time t aboard the three stations. However, general relativity taught us that the physical time scales are the proper time scales $\overset{1}{\tau}$, $\overset{2}{\tau}$ and $\overset{3}{\tau}$; that is the time beaten by the clocks aboard spacecraft 1, 2 and 3 respectively, *if* those are *not* constrained/synchronized[2]. Strictly speaking, $\overset{1}{\tau}$, $\overset{2}{\tau}$ and $\overset{3}{\tau}$ are different time scales which are not directly compatible, such as apples and oranges. The natural common language between those time scales is a time coordinate t such as the one associated with the Barycentric Coordinate Reference System, called TCB. In the LISA mission, the events, which should thus be recorded in the corresponding proper time scale aboard a given LISA spacecraft k, are the different emission or reception event at that spacecraft. The locally measured frequency shifts (against the local proper frequency) should also be regularly recorded on board according to the local proper time scale (see figure 1). Each spacecraft k data-record should thus

[2] Indeed, equation (7) for stations $k = 1, \ldots, 3$ orbiting the Sun is similar to the GPS clock synchronization [1] applied to clocks aboard the GPS constellation orbiting the Earth.

contain $\overset{k}{\tau}_{n\,\mathrm{em}\,k}$, $\overset{k}{\tau}_{n\,\mathrm{rec}\,k}$,

$$\Gamma^{jki}(\overset{k}{\tau}) \equiv \Gamma^{jki}|_{\overset{k}{\tau},\ \text{at } k \text{ with LF,OB,F,PM noises}} \tag{17}$$

and

$$\Lambda^{jk}(\overset{k}{\tau}) \equiv \Lambda^{jk}|_{\overset{k}{\tau},\ \text{at } k \text{ with LF,Q,OB,PM noises + GW signal + G}} \tag{18}$$

where we have adopted the notation $\overset{k}{\tau}_{\mathrm{em/rec}\,k}$ for the nth emission/reception event at spacecraft k on the time scale $\overset{k}{\tau}$. Consequently, to apply the TDI analysis, i.e. to compute (15) from (9), (10), (11), (12), we need to use equation (7) to convert from time scale $\overset{k}{\tau}$, in which the observed/simulated frequency shifts (17), (18) should be recorded, to time scale t.

To summarize this section, a coherent general relativistic approach of TDI analysis is necessary. This means general relativistic modelling of laser links [2] (that is t_{ij}, since geometric frequency shifts can be omitted owing to LISA frequency bandwidth), LISA station orbitography [12] and time scales, which is the subject of the present paper.

4. LISACode: a LISA simulator

A LISA simulator aims at an intimate understanding of the mission, providing an overview of data analysis. This means that the simulator will be a precious tool to find and test strategies to detect GW, to test the specificities of interferometric configurations for that purpose, to test the response of the detector (see, for example, LISA SIMULATOR [4], SYNTHETIC LISA [16] or LISACode [11]).

However, before getting to detection strategies, we have seen that the data must go through the TDI pre-processing analysis in order to remove (most of) LF and OB noises. A sine-qua-non condition for the LISA mission to work is for the TDI method to be efficient. Limitations on the effectiveness of the TDI technique come from the fact that the considered TDI generation is more or less adequate to the real orbitography and to the real laser links in the LISA mission. They also come from secondary noises (Q, F, PM, USO: Ultra Stable Oscillator(s), that is the clock aboard LISA spacecraft . . .) affecting the measurement, and certainly from the finite accuracy/precision of the quantities required to construct the corresponding TDI observables (see [14, section V]).

Consequently, one aim of LISACode is also to test TDI pre-processing efficiency for a realistic model of the LISA mission (orbitography, laser links, time scales) that is a coherent general relativistic rather than classical model, in the presence of realistic noises: LF, OB, Q, F, PM, USO. In that respect, the question that LISACode addresses is: is the residual LF plus OB noise left in a given TDI-generation observable acceptable? Presently, LISACode uses a classical orbitography model (including constellation breathing plus rotation around its centre of mass and around the barycentre), such as described in [6] and allows also for a static LISA or a rigidly rotating LISA. LISACode adopts a general relativistic laser links description according to [2] to compute t_{ij}. LISACode also contains LF, OB, PM and USO (shift and drift) noises; standard TDI observables of first and second TDI generation, as well as a procedure to compute arbitrary TDI observables. Each of the three USO noises *initially* represents *intrinsic* offset, difference in rate and drift of the clocks aboard one of the three LISA spacecraft. But via this USO-noise input in LISACode, the proper time versus TCB time transformations, $\overset{k}{\Delta}$, described in section 2.2, can also be introduced. There will then be interesting questions to answer: How will the precision of LISA ephemerides, used in LISACode to compute both

t_{ij} and $\overset{k}{\Delta}$, affect the efficiency of a given TDI generation? And how will the constraints on LISA ephemerides accuracy derived with LISACode compare with the theoretical constraints provided by Tinto *et al* (about 16 m for arm-length [14, section V A, (29) and (33)] and about 50 ns for clock synchronization [14, section V B, (39) and (40)]) for the 1.5th-generation standard TDI observables?

5. Conclusions

We provided the relativistic analytical and numerical estimates for spacecraft proper times versus coordinate (BCRS) time transformation, for a classical orbitography in the gravitational field of the Sun. We showed that the difference between station proper time and BCRS time reaches about 0.5 s over a one-year mission and exhibits an oscillatory trend with a maximal amplitude of about 0.0014 s. The difference in rate of station proper time versus BCRS time is of the order of 1.5×10^{-8}.

We then recalled how this time transformation fits in a general relativistic approach of time delay interferometry, necessary to lower the laser frequency noise and optical bench noise down to the gravitational wave detection threshold.

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