Shapiro delay of asteroids on LISA

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Abstract

In this paper, we examine the Shapiro delay caused by the close approach of an asteroid to the LISA constellation. We find that the probability that such an event occurs at a detectable level during the time interval of the mission is smaller than 1 %.

A mettre/changer pour la nouvelle version :

* Ref 1 :

a - "...without regard to the frequency content..."

Si ! C'est justement la condition sur la durée du passage qui assure que le fondamental (dans la TF du signal) se trouve bien dans la bande LISA !

—> mettre une phrase ds le texte, ou rendre l'existant plus explicite

b - shape of pseudo GW signal : peut-être suffit-il en fait de donner $h(t)$?

—> Il me semble qu'obtenir $h_{00}(t)$ est immédiat (c'est le potentiel gravitationnel (newtonnien) de l'asteroide). Peut-être aussi pour les autres composantes. Je vais regarder ce point. A vue de nez, la métrique de Schwarzschild, vue par un observateur en mouvement, donne peut-être la réponse ....

c - les simulations de Tania : je propose de faire une phrase dans le texte, renvoyant à une annexe ou on explique un peu ce qui est fait, dans quel but, et justifier ce qui en est conclu. A mettre d'ailleurs en lien avec le point (a) ci-dessus. Ceci en disant explicitement que si un/les referre(s) et/ou l'éditeur trouve(nt) que ce n'est pas indispensable, on pourra le retirer sans pb.

d - revoir certains points de l'introduction, relatifs à la description/utilité de ma mission.

* Ref 2 :

e - comme seuls les gros asteroides sont susceptibles d'être concernés, les trajectoires correspondantes sont parfaitement connues, et "the resulting event would easily be vetoed".

—> Argument très fort à mettre, qui d'ailleurs justifie qu'il n'est peut-être pas très utile de s'acharner sur des templates !!!

f - résultat d'intérêt faible : OK, mais l'estimation est tout de même utile, car l'existence d'évenements dans la bande de fréquence montre qu'il faut une étude dédiée pour savoir précisément à quoi s'en tenir.

—> à insérer dans la conclusion

g - faire l'étude statistique relative à l'effet étudié par Jean-Yves : tout à fait OK, j'y avais d'ailleurs pensé. Donc à mettre.

—> à insérer dans la conclusion (ou faire un chapitre "Discussion" dédié, où on peut aussi insérer le point (e))

I - Introduction

LISA is a space experiment devoted to the detection of gravitational waves, in the $\left[\sim 10^{-4}, \sim 10^{-1}\right]$ Hz frequency domain. LISA is a nearly equilateral triangular constellation of three spacecraft, located on an Earth orbit (about 20 degrees behind Earth), the precise inter distances of which are regularly measured by laser links [1]. The distances between the spacecraft depend on time, due to the gravitational field of the Sun [2] and planets, to be modeled in the frequency domain. Gravitational waves crossing the constellation will induce additional changes in distances. Then, tracking the time-dependency of the distances between spacecraft gives information on astrophysical sources of gravitational waves, and hence provides another useful way to explore the universe.

The gravitational field of the solar system is modeled including the Sun [2] and planets only. However, it is known that the terrestrial orbit is frequently crossed by asteroids, refered to as GC (geo-cruisers) in the following [3,4]. Since LISA is on an Earth orbit, close encounters with GCs will occur, and the gravitational field of the GC can generate a signal in LISA, which has to be distinguished from the expected extra-solar system gravitational wave signal to be tracked.

In a recent paper, Vinet [5] has examined the direct action of a GC on LISA constellation, i.e. the shift in position of a station by the gravitational field of the asteroid. He finds that it leads to a measurable signal if the involved GC passes sufficiently close to one of the spacecraft (and then far from the two others). In this paper, we are interested in another aspect of the problem. If a GC passes far from each spacecraft, but close to the segment joining two of them, the GC gravitational field can affect the measured distance between these two stations by Shapiro delay on the laser light used for that very measurement. The aim of this study is to examine if such an (impulsional-like) event is likely to occur. We find that the probability of the occurence of such an event at a detectable level, during the time interval of the mission, is quite negligible.

II - Condition for an asteroid encounter to cause a relevant Shapiro delay

Let us consider an asteroid passing close to the light beam linking two LISA spacecraft A and B. The space-time geometry in which the beam propagates can be formally written as

$$g_{\alpha\beta} = \eta_{\alpha\beta} + h_{\alpha\beta}^{(\mathrm{S.S.})} + h_{\alpha\beta}^{(\mathrm{ast.})} + h_{\alpha\beta}^{(\mathrm{G.W.})}$$

where $\eta_{\alpha\beta} = diag\left(-1, +1, +1, +1\right)$ is the Minkowsky metric, $h_{\alpha\beta}^{(\mathrm{S.S.})}$ the gravitational field due to the Sun and planets, $h_{\alpha\beta}^{(\mathrm{ast.})}$ the field due to the close asteroid, and $h_{\alpha\beta}^{(\mathrm{G.W.})}$ the field of the gravitational wave. The gravitational wave term induces a change in the distance between the two stations of the order of

$$\delta L^{(\mathrm{G.W.})} \sim hL$$

where h is the amplitude of $h_{\alpha\beta}^{(\mathrm{G.W.})}$, and L, the distance between A and B. On the other hand, $h_{\alpha\beta}^{(\mathrm{ast.})}$ is of order $2Gm/(rc^2)$, where m is the asteroid mass. The close approach induces a Shapiro time delay δt in the flight time of the photon, hence a change in the distance of the order of [6]

$$\delta l = c.\delta t \approx \frac{4Gm}{c^2} \ln \left(\frac{4 r_A r_B}{\Delta^2} \right),$$

where r_A (resp. r_B) is the distance between the asteroid and spacecraft A (resp. B) and Δ the distance between the asteroid and the segment joining the spacecraft A and B. Let b be the impact parameter of the encounter (minimum value of Δ during the approach). In the case where $b << L$ (we will find that it is a necessary condition for the signal to be observable), it is easy to see that the maximum possible value for δl satisfies

$$\delta L^{(\text{ast.})} \lesssim \frac{8Gm}{c^2} \ln \frac{L}{b}.$$

If $H_{\min}$ is the smallest value of h accessible to the experiment, the asteroid generates a detectable gravitational signal only if

$$\frac{8Gm}{c^2} \ln \frac{L}{b} \gtrsim H_{\min} L.$$

Let ρ and D be the density and the (mean) diameter of the considered asteroid respectively. The above unequality leads to the following condition on the impact parameter

$$b \lesssim L \exp \left\{ -\frac{3}{4\pi} \frac{c^2}{G \rho D^3} H_{\min} L \right\} \tag{0.1}$$

for the asteroid signal to be observable. This gives, numerically,

$$b \lesssim \left(5.10^6 \text{ km}\right) \exp \left\{ -8000. \frac{H_{\min}}{10^{-20}} \left(\frac{\rho}{2 \text{ g/cm}^3} \right)^{-1} \left(\frac{D}{1 \text{ km}} \right)^{-3} \right\} \tag{0.2}$$

where we have taken $L = 5.10^6$ km, the average inter-distance between spacecraft. Let us consider an asteroid of 10 km in diameter (resp. 15 km). One finds (with $\rho = 2$ g/cm^3 and $H_{\min} = 10^{-20}$) $b \lesssim 1700$ km (resp. 470000 km). We note that for a 8 km diameter asteroid, the impact parameter should be smaller than 1 km, that is smaller than the asteroidal radius, which means that the beam would be occulted.

The characteristic time of the encounter is given by $\tau \sim b/V$, where V is the relative velocity of the asteroid. Extensive simulations from fictitious impulsive signals, the duration of which ranging from 1 one to 10^5 s, confirm that $H_{\min}$ is always $\geq 10^{-20}$, and that the detection is not efficient outside the interval $\left[10\ s, 10^4\ s\right]$ (see appendix I). Since one should have τ in the time interval $\left[10\ s, 10^4\ s\right]$, b has to satisfy the additional condition

$$(10\ s).V \lesssim b \lesssim (10^4\ s).V \tag{0.3}$$

besides condition (0.2). Since $V \sim 15$ km/s, one sees that only a small number of asteroids will effectively be relevant to LISA at $H_{\min} = 10^{-20}$, verifying both conditions (0.2) and (0.3). Indeed, only asteroids larger than 9 km in diameter can generate a signal at a detectable level with characteristic encounter times in this interval, but only about 15 GCs are larger than 9 km in diameter [4]. If one takes $\rho = 2.7$ g/cm^3, this limit in diameter becomes 8 km, and about 20 GCs are larger than 8 km in diameter.

III - Probability of a relevant encounter

Let $n(\geq D_0)$ be the mean number density of GCs with a diameter $D \geq D_0$ in the neighbourhood of the Earth orbit. Let V be the mean relative velocity of GCs and the Earth. The number of GCs, of diameter larger than D_0, passing at a distance between b and $b+db$ from the segment [A,B] (with $b << L$) during a time interval dt, is of the order of $2n(\geq D_0).L.db.V.dt$. Let T be the duration of the LISA mission. From eq. (0.1), the condition of detectability by LISA is $D \geq D_0$, with

$$D_0^3 = \frac{3}{4\pi}\frac{c^2}{G\rho}\frac{HL}{\ln(L/b)}.$$

The number E of events observed during the duration of the mission is then of the order of

$$E \sim 6LVT \int_{b_{\min}}^{b_{\max}} n(D \geq D_0)\, db$$

since there are three arms in the LISA configuration. The lower and upper bounds $b_{\min}$ and $b_{\max}$ are the minimal and maximal values of b, related to the LISA frequency sensitivity curve for impulsive events (0.3). The total number of GCs is estimated to be [4]

$$N(\geq D_0) \sim 1090 \left(\frac{D_0}{1\ km}\right)^{-1.95}. \qquad (0.4)$$

To evaluate E, only the density number in the vicinity of Earth is needed, not N, the total numbers of GCs. From (0.4) and the estimation obtained in the appendix II, the mean number of GCs per unit volume (per $(AU)^3$) in the vicinity of Earth orbit is

$$n(\geq D_0) \sim 94 \left(\frac{D_0}{1\ km}\right)^{-1.95}.$$

Then

$$E \sim \frac{0.51}{(1\ A.U.)} \int_{b_{\min}}^{b_{\max}} \left[\frac{\rho}{2\ g/cm^3}\left(\frac{H_{\min}}{10^{-20}}\right)^{-1} \ln\frac{L}{b}\right]^{0.65} db.$$

We have taken $V = 15\ km/s$ and $T = 3\ yrs$. The values of $b_{\min}$ and $b_{\max}$ were given in (0.3), and $\ln(L/b)$ in the integral varies in the interval $[\sim 3.5; \sim 10.4]$. The minimal amplitude $H_{\min}$ depends on the characteristic time of the encounter, then depends on b, but, as stated before, it can be bounded by 10^{-20}. Since ρ is always of the order of $2\ g/cm^3$ (for asteroids, it belongs to the interval $\left[1.3\ g/cm^3; 2.7\ g/cm^3\right]$), the number of relevant events during the LISA mission is bounded by

$$E \lesssim 1.65\ 10^{-3}$$

which means that the probability to observe one event caused by a Shapiro delay related to a close GC approach is quite negligible. This number (probability) becomes $2\ 10^{-3}$ if one takes $\rho = 2.7\ g/cm^3$.

IV - Conclusion

The present study shows that the possibility of detecting an asteroid through Shapiro delay by the LISA mission :

- concerns only a small number of GCs (about twenty at best). An occultation of the laser beam occurs before the detection condition is satisfied for GCs with a diameter less than $\sim 8\ km$;

- has a very low probability to occur during the time interval of the mission, at best of the order of some 10^{-3}.

Appendix I : Minimal detectable amplitude

The signal to noise ratio averaged over all sky directions and polarizations can be expressed as [7]

$$\left(\frac{S}{N}\right)^2 = 2\int_0^\infty d\nu \frac{S_h(\nu)}{S_{eff}(\nu)}$$

where $S_{eff}(\nu)$ is the effective sensitivity of LISA. In our calculations, we adopt the position noise budget for a standard Michelson configuration, including the contribution of the galactic binary WD-WD confusion noise [8].

Let $h(t) = Hf(t)$ be a gravitational signal of duration T and of amplitude H (the function $f(t)$ being of amplitude unity). The corresponding spectral density $S_h(\nu)$ can be expressed as

$$S_h(\nu) = H^2 \left|\widetilde{f}(\nu, T)\right|^2$$

$\widetilde{f}$ being the Fourier transform of f.

Combining the above equations, one obtains for the minimal detectable amplitude

$$H_{\min} = \frac{(S/N)_{\min}}{2\sqrt{I}}$$

where

$$I = \int_0^\infty d\nu \frac{\left|\widetilde{f}(\nu, T)\right|^2}{S_{eff}(\nu)}.$$

Following the convention adopted in the LISA community, we assumed a detectability threshold of $(S/N)_{\min} = 5$.

Appendix II : From total asteroid distribution to volumic distribution near Earth orbit

The distribution of GCs with respect to orbital elements (a, e, i) is given in reference [3]. Since one is interested in an order of magnitude estimate rather than in precise results, let us make the following hypotheses and simplifications :

- the diameter distribution of GCs is independent of the orbital-element distributions ;

- the distribution in inclination is limited to the interval $[0, i_{\max}]$, in which it is uniform.

Let us consider an asteroid with orbital elements a and e. The probability that this asteroid is at a distance from the sun in the interval $[r, r + dr]$, at an arbitrary time, is given by $dP\left(r,r+dr\right)=2dr/\left(\theta\left|\dot{r}\right|\right)$, where θ is the period. Then, $\dot{r}$ is given by the energy integral, and one finds

$$dP\left(r,r+dr\right)=\frac{dr}{\pi a\sqrt{\left(e\frac{a}{r}\right)^{2}-\left(1-\frac{a}{r}\right)^{2}}}.$$

Let $p\left(a,e\right)$ be the density distribution in a and e, so that $p\left(a,e\right)dade$ is the probability that a GC, arbitrarily chosen in the population, has its semi-major axis and eccentricity in the intervals $[a,a+da]$ and $[e,e+de]$, respectively. Using the hypothesis on inclination distribution, one finds that the number density of GCs on an Earth orbit is related to the total population N by

$$n=\frac{N}{4\pi^{2}\sin i_{\max}}\int_{\{a,e\}}\frac{p\left(a,e\right)dade}{a\sqrt{e^{2}a^{2}-\left(1-a\right)^{2}}}$$

where one has replaced r by unity (1 $A.U.$). In this expression, a is expressed in $A.U.$, n in $\left(A.U.\right)^{-3}$ and $\{a,e\}$ is the integration domain in the $a-e$ plane. From [3], let us consider that the integration domain is bounded by $0.5<a<3$ and $0.2<e<1$. Besides, for an asteroid to be a GC, one has necessarily $a\left(1-e\right)<1<a\left(1+e\right)$. Since one is only interested in an order of magnitude, let us repace $p\left(a,e\right)$ by its mean value $\left\langle p\left(a,e\right)\right\rangle=\left(1+\ln 1.44\right)^{-1}\sim 0.7328$ in the integral. One finds

$$n=N\frac{\left\langle p\right\rangle K}{4\pi^{2}\sin i_{\max}}$$

with $K=3\pi/10+\arcsin\left(2/3\right)+2/3\ln\left[\left(3+\sqrt{5}\right)/2\right]\sim 2.3138$. From [3], a reasonnable value for $i_{\max}$ is 30 degrees. This leads to

$$n\left(\geq D_{0}\right)\sim 0.086N\left(\geq D_{0}\right).$$

This is the mean density (per $\left(A.U.\right)^{3}$) of GCs with diameter $D\geq D_{0}$, at one astronomical unit from the Sun.

Acknowledgements

We are particularly grateful to P. Michel and A. Morbidelli, from Observatoire de la Côte d'Azur, and J. S. Stuart, from MIT Lincoln Laboratory, who provided us with useful information on GCs.

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